Joint Power and Data Allocation in Multi-Carrier Full-Duplex Relaying Networks Operating with Finite Blocklength Codes

Xiaopeng Yuan, Hao Jiang, Yulin Hu, *Senior Member, IEEE*, Bo Li,

Eduard Jorswieck, *Fellow, IEEE*, and Anke Schmeink, *Senior Member, IEEE*

Abstract

In this paper, we study a full-duplex (FD) relaying network operating with finite blocklength (FBL) codes. Based on Polyanskiy's FBL model, we characterize the FBL reliability of the relaying network under both decode-and-forward (DF) and amplify-and-forward (AF) relaying schemes. Based on the characterisation, we provide reliability-optimal designs via optimal power allocation for both schemes in a single-carrier scenario. In particular, we prove that under the FD DF relaying scheme the (tightly approximated) overall error probability is convex in the transmit power at the relay. In addition, we show that minimizing the overall error probability of the FD AF relaying is equivalent to maximizing the overall signal to interference plus noise ratio (SINR), which is further proved to be pseudo-concave. Then, the designs for a single-carrier scenario are further extended to a multi-carrier scenario with a joint power and data allocation among carriers. In particular, for either the FD DF or FD AF relaying scheme, a joint optimization problem is reformulated to a single problem maximizing the reliability via finding and achieving the optimal SINRs, while auxiliary variables are introduced in FD AF relaying to facilitate the reformulation. Based on mathematical analysis, we respectively construct convex approximations and subsequently propose iterative algorithms, with which the error probability is reduced iteratively until an eventual convergence to an efficient suboptimal value. Hence, a corresponding suboptimal data and power allocation solution can be constructed for the multi-carrier scenario. Via numerical analysis, we validate our analytical model and the proposed allocation algorithms. The FD DF and FD AF relaying schemes are compared with direct transmission in both single-carrier and multi-carrier scenarios, and the benefits of applying FD relaying schemes and joint optimization among multiple carriers are observed.

Index Terms

Finite blocklength, full-duplex relaying, error probability, resource allocation, multiple carriers.

I. INTRODUCTION

In the design of future wireless networks, low latency and high reliability are two key merits to enable delay-sensitive and mission-critical applications of next-generation wireless networks [2], [3], such as autonomous driving, virtual/augmented reality (VR/AR), remote surgery and industrial automation [4]–[6]. In the fifth generation (5G) and sixth generation (6G), this concept is called ultra-reliable low-latency communication (URLLC). For example, according to the recent research article [7] and standard in 3GPP [8], URLLC applications, such as factory automation and VR/AR, generally require a delay bound of 1–10 ms and a packet error probability of 10^{-5}–10^{-7}, while the requirements for remote surgery are preferred to be even more critical.

Based on the transmitted data sizes, we distinguish two URLLC applications. One is the typical short packet transmission, in which the data size is generally small. As a typical scenario, a reporting sensor continuously reports time-sensitive information of several bits describing the state of the target, e.g, traffic, machine working states and so on. For such applications with a relatively lower data rate requirement, researchers have contributed to improve the performance with respect to reliability [9] and energy efficiency [10], [11]. The second type of the applications has relatively larger data sizes, e.g., the VR/AR or remote surgery scenarios with real-time video streams. In other words, due to the latency constraint and frequent update demand, these applications require the network to provide URLLC transmissions while supporting relatively higher (equivalent) date rates, which makes the system design more challenging. In fact, with a given latency constraint and a high coding rate target, a system could anyway send out these data before the delay deadline. But the high reliability of such transmission is extremely difficult to be guaranteed.

Fortunately, there exist many reliability enhancement technologies. Among them, relaying has been proved to be an effective way to provide significant improvements on the transmission reliability [12] and supporting a relatively higher coding rate [13], [14]. In comparison to the direct wireless communications, by applying an additional transceiver as a relay, the destination may receive a much stronger signal which is received by the relay from source node and then forwarded to the destination through a possibly better wireless channel. As for the relaying technologies, there are two main forwarding schemes [15], i.e., decode-and-forward (DF) scheme and amplify-and-forward (AF) scheme. By applying a DF scheme, the relay first decodes the received signal and then forwards a re-encoded one if the decoding process is successful, while

it forwards nothing when the decoding process fails. On the other hand, an AF relaying scheme directly scales the received signal without decoding, where the destination always receives a signal from the relay.

Typically, a relay node is operated in a half-duplex (HD) mode [12]–[15],where one from the source and relay is idle when the other one is performing transmission, i.e., the source and relay transmit one at a time. By implementing the advanced self-interference cancellation (SIC) techniques [16], [17], full-duplex (FD) relaying is capable of operating transmission and reception simultaneously. As a result, within the same time block, deploying an FD relaying significantly improves the throughput [18]–[20]. From another point of view, when the target of throughput or data rate are given, HD relaying needs to set a doubled data rate at each hop to achieve the equivalent end-to-end data rate target. Differently, FD relaying just needs to set the data rate of each hop same as the target, which is expected to have a better reliability. Moreover, by exploiting multiple subcarries for the two hops of the FD relaying and performing resource allocation among these subcarries, the above advantages of deploying DF relaying with respect to throughput and reliability can be further promoted [21], [22].

However, existing studies on the FD relaying are conducted based on an ideal assumption of communicating arbitrarily reliably at Shannon's capacity, which is only true when the code blocklengths become infinitely long, i.e., in the so-called infinite blocklength (IBL) regime. Unfortunately, in a practical system the codes can only have finite blocklengths (FBL). In particular, when the network is required to operate under a low-latency constraint, the coding blocklengths are required to be short. Hence, it is essential to study the FBL performance while explicitly taking into account decoding error probabilities in the performance analysis and system design of networks supporting URLLC applications. The authors in [23] have rigorously characterized the relationship in the FBL regime among the blocklength, achievable coding rate and error probability. Moreover, the system performance of an HD relaying network within FBL regime has also been analytically studied in [24], [25]. The FBL throughput in both HD and FD relaying networks has been recently approximately discussed in [26], where the Q-function in the FBL error probability model is approximated by a linear function. However, to the best of our knowledge, a fundamental analytical study on FBL performance characterization of the FD relaying schemes (including and comparing AF and DF relaying schemes) as well as the resource allocation among multiple carriers aiming at maximizing the reliability while supporting

high data rate and low latency transmissions, is still missing.

In this work, we consider an FD relaying network, where the transmissions are operating with FBL codes due to the low latency requirements. We characterize the FBL performance and provide joint power and data allocation designs for such network, where both DF relaying and AF relaying schemes are considered. The main contributions of our work are listed as follows:

- **FBL Reliability Characterisation**: We characterize the reliability model for an FD relaying network in FBL regime. The error probabilities under both DF and AF relaying schemes are derived.

- **Power allocation in Single-Carrier Scenario**: For a single-carrier scenario, an optimal power allocation design between the two hops of the FD relaying is provided for both DF and AF scenarios. In particular, we prove the convexity of the power allocation problem in the FD DF relaying case and the pseudo-concavity of signal-to-interference-plus-noise ratio (SINR) in FD AF relaying, respectively, with which the optimal solution of the power allocation problem can be efficiently obtained.

- **Power and Data allocation in Multi-Carrier Scenario**: For multi-carrier scenario, a joint power and data allocation among carriers is considered. We first rigorously prove the joint convexity of the error probability with respect to SINR and data size, which can largely assist the related system analysis in FBL regime. Then, we propose iterative algorithms for FD DF and FD AF relaying cases, which have resulted in efficient suboptimal solutions.

- **Numerical Investigation:** Finally, the characterized FBL reliability model and the proposed algorithms are validated via numerical analysis. In addition, via a set of comparisons, we confirm the benefits of deploying FD relaying, and the advantages of jointly allocating resource among carriers.

The organization of the paper is as follows. We first introduce our considered FD relaying system and review the FBL performance model in Section II. Then, in Section III, considering a single-carrier scenario and a power constraint, we study the reliability performance of the FD relaying network under DF and AF relaying schemes. After that, the work is extended into multi-carrier scenario in Section IV and two iterative algorithms are correspondingly proposed. Finally, our work is numerically evaluated in Section V and concluded in Section VI.
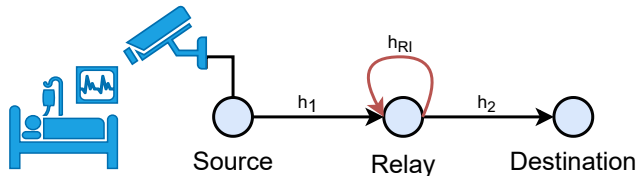
Fig. 1. Example of the considered two-hop FD relaying scenario.

II. Preliminaries

In this section, we first introduce our FD relaying system model under both the DF and AF relaying schemes. Subsequently, the FBL performance model is reviewed.

A. System model

We focus on a simple two-hop network with a source node S, a destination node D and an FD relay R as schematically displayed in Fig. 1. The data packet of D bits generated by source node is assumed to be relatively large. In Fig. 1, we show an example scenario of remote surgery, in which the video stream (with a relatively large data packet) is required to be frequently updated in time. We assume the relatively large attenuation between S and D forces the data transmission from S to D to be operated via two hops, namely from S to R then from R to D. To support this relatively high end-to-end (from S to D) data rate, an FD relaying is applied, which is capable of reducing data rate on each link and correspondingly enhancing the transmission reliability, in comparison to HD relaying. In addition, the transmission of a packet has a fixed end-to-end latency constraint. And we assume that the data packet is immediately transmitted after being generated and will be dropped when the latency constraint is violated (i.e., assuming the latency is more important than missing a short slide of the video stream). In other words, the data packet has no queuing delay. At the same time, we assume the other delays in addition to the two-hop transmission, e.g., due to data generating, processing and decoding, are constant (or upper-bounded). Hence, the maximal allowed total blocklength, denoted by M, for the two-hop relaying can be determined. Finally, we consider an ultra-reliable transmission scenario [27] where the error probability of the data transmission via the two-hop relaying network should be (much) lower than 10^{-1}.

Furthermore, both the DF and AF relaying schemes are considered. Accordingly, the resulting different frame structures are shown in Fig. 2. For FD DF relaying, the relay will wait for the complete reception of all the symbols from source for a successful decoding and then forward the re-encoded data packet to the destination subsequently. Due to the existence of a necessary decoding process, the signal reception and forwarding cannot be operated simultaneously.
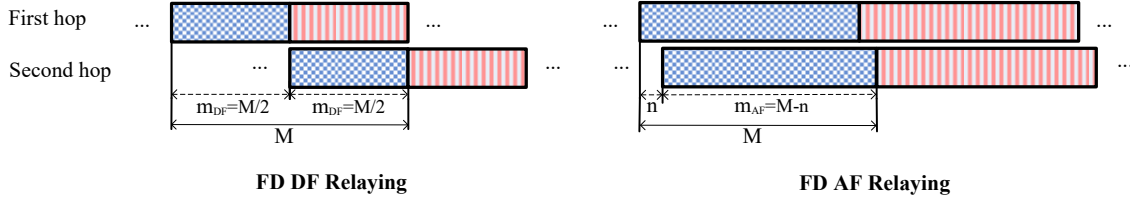
Fig. 2. Frame structures of the considered FD DF and FD AF relaying scenarios.

Therefore, the total blocklength M is equally divided by a blocklength m_{DF}, which satisfies $2m_{\mathrm{DF}} = M$, for each relaying hop under FD DF scheme. On the other hand, the relay under FD AF scheme allows directly amplifying and forwarding the received radio frequency (RF) signal from source to the destination. Hence, the received symbols can be immediately forwarded without any decoding process. We denote by n the smallest amount of symbols that can be recognized and forwarded by the FD AF relay. As a result, the blocklengths for both relaying hops are equal to $m_{\mathrm{AF}} = M - n$, as shown in Fig. 2. For an ideal case where each symbol can be immediately detected and forwarded by FD AF relay, the forwarding process will operate per symbol, i.e., $n = 1$. In addition, in FD relaying, the AF scheme also induces deteriorated received SINR which is caused by the direct amplification of self-interference and noise [28].

B. Channel Model

In particular, we start with the single-carrier scenario to facilitate the power allocation in Section III. After that, the topology and channel model will be directly extended to multi-carrier scenario, as performed in Section IV. Here, we denote by h_1 and h_2 the channel coefficients of the S-R backhaul link and R-D relaying link, and denote by L_i and z_i the gains of the path-loss and the channel fading. Then, we model the channel gain as $|h_i|^2 = L_i z_i, i = 1, 2$. Note that in the considered low-latency scenario, the period of the two-hop transmission is significantly short in comparison to the channel coherence time, so that the fading gains can be treated as a constant within a transmission period. In addition, due to imperfection of self-interference cancellation in FD techniques, the loop interference at the FD relay is significantly but not completely reduced. we denote by h_{RI} the residual loop interference at the relay. We assume perfect channel state information (CSI) at the receivers and in particular at the source. The total power budget/constraint for transmitting a data packet in a transmission period is p_t, while the transmit power at the source and the relay are denoted by p_{S} and p_{R}, respectively.

For both DF and AF schemes, the expressions of the received signals at the relay are the same as

$$y_1 = \sqrt{p_{\mathrm{S}}} h_1 x + \sqrt{p_{\mathrm{R}}} h_{\mathrm{RI}} x_{\mathrm{RI}} + w_1, \tag{1}$$

where x is the transmitted signal from S and x_{RI} is the self-interference signal from R, both of which have a unit power. In addition, w_1 represents the additive white Gaussian noise (AWGN)

in the S-R link with power σ_1^2. Then, the signal to SINR at the relay is described as

$$\gamma_1 = \frac{p_{\mathrm{S}}|h_1|^2}{p_{\mathrm{R}}|h_{\mathrm{RI}}|^2 + \sigma_1^2}. \tag{2}$$

On the other hand, the received signals at destination have different expressions in the two schemes. In particular, the received signal at the destination under the DF scheme is given by[1]

$$y_{\mathrm{DF},2} = \sqrt{p_{\mathrm{R}}}h_2 x + w_2, \tag{3}$$

where w_2 is the AWGN in the R-D link with power σ_2^2, resulting signal-to-noise ratio (SNR)

$$\gamma_{\mathrm{DF},2} = \frac{p_{\mathrm{R}}|h_2|^2}{\sigma_2^2}. \tag{4}$$

For the FD AF relaying, y_1 is simply amplified and forwarded, i.e., resulting in an incidentally amplified residual self-interference and noise. The received at destination is thus given by

$$y_{\mathrm{AF},2} = \left(\sqrt{p_{\mathrm{S}}}h_1 x + \sqrt{p_{\mathrm{R}}}h_{\mathrm{RI}}x_{\mathrm{RI}} + w_1\right)\sqrt{Gp_{\mathrm{R}}}h_2 + w_2, \tag{5}$$

where $G = \frac{1}{p_{\mathrm{S}}|h_1|^2 + p_{\mathrm{R}}|h_{\mathrm{RI}}|^2 + \sigma_1^2}$ is the amplifier gain. Then, the received SINR at the destination is

$$\gamma_{\mathrm{AF},2} = \frac{Gp_{\mathrm{S}}p_{\mathrm{R}}|h_1|^2|h_2|^2}{Gp_{\mathrm{R}}^2|h_{\mathrm{RI}}|^2|h_2|^2 + Gp_{\mathrm{R}}|h_2|^2\sigma_1^2 + \sigma_2^2} = \frac{p_{\mathrm{S}}p_{\mathrm{R}}|h_1|^2|h_2|^2}{p_{\mathrm{R}}^2|h_{\mathrm{RI}}|^2|h_2|^2 + p_{\mathrm{R}}|h_2|^2\sigma_1^2 + \sigma_2^2(p_{\mathrm{S}}|h_1|^2 + p_{\mathrm{R}}|h_{\mathrm{RI}}|^2 + \sigma_1^2)}. \tag{6}$$

C. The FBL performance model

The FBL performance has been analyzed in [23] via applying the normal approximation. Later on, the third-order term in the normal approximation is further addressed in [29] especially for the AWGN channel, which leads to a higher performance accuracy. For an AWGN channel, the coding rate r (in bits per channel use) with error probability $0 < \varepsilon < 1$, SNR γ, and blocklength m is shown to have the following asymptotic expression [29]:

$$r = \mathcal{R}(\gamma, \varepsilon, m) \approx \mathcal{C}(\gamma) - \sqrt{\frac{V(\gamma)}{m}}Q^{-1}(\varepsilon) + \frac{\log_2 m}{m}, \tag{7}$$

with the Shannon capacity $\mathcal{C}(\gamma) = \log_2(1 + \gamma)$, channel dispersion $V(\gamma) = \frac{\gamma(\gamma+2)}{(\gamma+1)^2}\log_2^2 e$, and Gaussian Q-function $Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}}e^{-t^2/2}dt$. By reformulating (7), the (block) error probability can be expressed as:

$$\varepsilon = \mathcal{P}(\gamma, r, m) \approx Q\left(\frac{\mathcal{C}(\gamma) + \frac{\log_2 m}{m} - r}{\sqrt{V(\gamma)/m}}\right). \tag{8}$$

As the FBL performance approximation with the third order term has been shown in [23], [29] to be tight when the blocklength is larger than 100. For simplicity, existing works, e.g., [30],

---

[1]It should be pointed out that the DF relay only forwards the data packet to the destination when it decodes the signal from the source successfully. Due to the FBL constraint, errors possibly appear in this step. The probability of the error will be later on discussed in Section III.

[31], usually assume the above approximations to hold with equality and exploiting them as the rate and error expressions. Following the same idea, in our analysis we assume $M \geq 200$ (which is reasonable for practical two-hop relaying systems) and apply the equality in studying the FBL performance of the FD DF and FD AF relaying schemes.

III. SINGLE CARRIER SCENARIO: POWER ALLOCATION FOR FBL RELIABILITY

In this section, we characterize the reliability models for both FD DF and FD AF relaying schemes. In particular, we minimize the overall error probability of the two-hop transmission by applying optimal power allocation under a given latency constraint M and a power consumption constraint Mp_t. We first discuss the FD DF relaying scheme and subsequently address the AF relaying case.

A. Achievable FBL Reliability of FD DF Relaying

According to (2) and (8), the decoding error probability at the FD DF relay is obtained by $\varepsilon_{\mathrm{DF},1} = \mathcal{P}\left(\gamma_1, \frac{2D}{M}, \frac{M}{2}\right)$. Similarly, the decoding error probability at the destination is given by $\varepsilon_{\mathrm{DF},2} = \mathcal{P}\left(\gamma_{\mathrm{DF},2}, \frac{2D}{M}, \frac{M}{2}\right)$. Hence, the overall error probability of transmitting a data packet via the two-hop FD DF relaying is given by

$$\varepsilon_{\mathrm{DF}} = \varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2} - \varepsilon_{\mathrm{DF},1}\varepsilon_{\mathrm{DF},2} \approx \varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2}, \tag{9}$$

while the approximation is tight due to the fact that $\varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2} \gg \varepsilon_{\mathrm{DF},1}\varepsilon_{\mathrm{DF},2}$ holds as we consider a high-reliability network with $\max\{\varepsilon_{\mathrm{DF},1}, \varepsilon_{\mathrm{DF},2}\} < \varepsilon_{\mathrm{DF}} \leq 10^{-1}$. In the following, we consider minimizing $\varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2}$ to obtain the achievable reliability of the FD DF scheme.

Obviously, $\varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2}$ is influenced by the choices of p_{S} and p_{R}. Note that under the power consumption constraint, we have $\frac{M}{2}p_{\mathrm{S}} + \frac{M}{2}p_{\mathrm{R}} = Mp_t$, i.e., $p_{\mathrm{S}} + p_{\mathrm{R}} = 2p_t$. Hence, the achievable reliability of the FD DF relaying can be obtained by solving the following optimization problem

$$\begin{aligned} \min_{p_{\mathrm{R}}} \quad & \varepsilon_{\mathrm{DF},1} + \varepsilon_{\mathrm{DF},2} \\ s.t. \quad & p_{\mathrm{S}} = 2p_t - p_{\mathrm{R}}, \\ & 0 < p_{\mathrm{R}} < 2p_t. \end{aligned} \tag{10}$$

To solve Problem (10), we provide the following proposition.

Proposition 1. *Considering an FD DF relaying network supporting a reliable transmission where target error probability $\varepsilon_{\mathrm{DF}} \leq \varepsilon_{\max}$, $M \geq M_{\min}$ and the SNR/SINR of each link $\gamma_i \geq 1$ hold, the objective of Problem (10) is convex in p_{R} when the relation*

$$\varepsilon_{\max} \leq Q\left(\frac{2\sqrt{2}}{0.37\sqrt{M_{\min}}}\right) \tag{11}$$

holds between $M_{\min}$ and $\varepsilon_{\max}$.

Proof. According to (9), we prove the proposition by showing $\frac{\partial^2 \varepsilon_{\mathrm{DF},i}}{\partial p_{\mathrm{R}}{}^2} \geq 0$ for link i, $\forall i \in \{1,2\}$. In particular, we first show the convexity of $\varepsilon_{\mathrm{DF},i}$ to γ_i by investigating $\frac{\partial^2 \varepsilon_{\mathrm{DF},i}}{\partial \gamma_i{}^2}$, and then based on the results discuss the sign of $\frac{\partial^2 \varepsilon_{\mathrm{DF},i}}{\partial p_{\mathrm{R}}{}^2}$.

According to (8), we have

$$\frac{\partial \varepsilon_{\mathrm{DF},i}}{\partial \gamma_i} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w_{\mathrm{DF},i}^2}{2}\right)\left(-\frac{\partial w_{\mathrm{DF},i}}{\partial \gamma_i}\right), \tag{12}$$

$$\frac{\partial^2 \varepsilon_{\mathrm{DF},i}}{\partial \gamma_i{}^2} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w_{\mathrm{DF},i}^2}{2}\right)\left(w_{\mathrm{DF},i}\left(\frac{\partial w_{\mathrm{DF},i}}{\partial \gamma_i}\right)^2 - \frac{\partial^2 w_{\mathrm{DF},i}}{\partial \gamma_i{}^2}\right), \tag{13}$$

where $w_{\mathrm{DF},i}(\gamma_i) = \frac{\mathcal{C}(\gamma_i) + \frac{\log_2(M/2)}{M/2} - 2D/M}{\sqrt{2V(\gamma_i)/M}}$. In addition, the first and second order derivatives of $w_{\mathrm{DF},i}$ to γ_i are respectively given by

$$\frac{\partial w_{\mathrm{DF},i}}{\partial \gamma_i} = \sqrt{\frac{M}{2}} \frac{\gamma_i(\gamma_i+2) - \left(\ln(1+\gamma_i) + \frac{\ln(M/2)}{M/2} - \frac{2D\ln 2}{M}\right)}{(\ln 2)^3 V(\gamma_i)^{\frac{3}{2}}(1+\gamma_i)^3} \geq \sqrt{\frac{M}{2}} \frac{\gamma_i(\gamma_i+2) - \ln(1+\gamma_i) - \frac{1}{e}}{(\ln 2)^3 V(\gamma_i)^{\frac{3}{2}}(1+\gamma_i)^3}. \tag{14}$$

$$\frac{\partial^2 w_{\mathrm{DF},i}}{\partial \gamma_i{}^2} = \sqrt{\frac{M}{2}} \frac{\left((1+\gamma_i) - \frac{1}{1+\gamma_i}\right)\left(2(1+\gamma_i) - \frac{1}{1+\gamma_i}\right) - 3\left(\gamma_i(1+\gamma_i) - \ln(1+\gamma_i) - \frac{\ln(M/2)}{M/2} + \frac{2D\ln 2}{M}\right)}{(\ln 2)^5 V(\gamma_i)^{\frac{5}{2}}(1+\gamma_i)^4}$$

$$= \sqrt{\frac{M}{2}} \frac{-(1+\gamma_i)^2 + \frac{1}{(1+\gamma_i)^2} + 3\left(\ln(1+\gamma_i) + \frac{\ln(M/2)}{M/2} - \frac{2D\ln 2}{M}\right)}{(\ln 2)^5 V(\gamma_i)^{\frac{5}{2}}(1+\gamma_i)^4} \tag{15}$$

$$\leq \sqrt{\frac{M}{2}} \frac{1}{(\ln 2)^5 V(\gamma_i)^{\frac{5}{2}}(1+\gamma_i)^4}\left[-(1+\gamma_i)^2 + \frac{1}{(1+\gamma_i)^2} + 3\ln(1+\gamma_i) + \frac{3}{e}\right].$$

where the inequalities hold due to the fact that $\frac{\ln(M/2)}{M/2} - \frac{2D\ln 2}{M} \leq \frac{\ln(M/2)}{M/2} \leq \frac{1}{e}$. Denote $f(x) = x^2 + 2x - \ln(1+x) - \frac{1}{e}$, $x \in [1, +\infty)$. Since $f(1) = 1.939 > 0$ and $f'(x) = 2x + 2 - 1/(1+x) > 0$ holds when $x \geq 1$, we have $f(x) \geq f(1) > 0$, $\forall x \in [1, +\infty)$. Applying this to (14), we have $\frac{\partial w_{\mathrm{DF},i}}{\partial \gamma_i} > 0$, $\forall \gamma_i \geq 1$. In addition, it can be also shown that $-(1+\gamma_i)^2 + \frac{1}{(1+\gamma_i)^2} + 3\ln(1+\gamma_i) + \frac{3}{e} < 0$ for $\gamma_i \geq 1$. Hence, $\frac{\partial^2 w_{\mathrm{DF},i}}{\partial \gamma_i{}^2} < 0$ holds, which results in $\frac{\partial^2 \varepsilon_{\mathrm{DF},i}}{\partial \gamma_i{}^2} > 0$.

So far, we have studied the relationship between $\varepsilon_{\mathrm{DF,i}}$ and γ_i. Next, let us take into account the relationship between γ_i and p_{R} for $i = 1,2$. When $i = 2$, it can be easily obtained from (4) that $\frac{\partial^2 \gamma_2}{\partial p_{\mathrm{R}}{}^2} = 0$. Accordingly, we have

$$\frac{\partial^2 \varepsilon_{\mathrm{DF},2}}{\partial p_{\mathrm{R}}{}^2} = \frac{\partial^2 \varepsilon_{\mathrm{DF},2}}{\partial \gamma_2{}^2}\left(\frac{\partial \gamma_2}{\partial p_{\mathrm{R}}}\right)^2 \geq 0. \tag{16}$$

When $i = 1$, $\frac{\partial^2 \varepsilon_{\mathrm{DF},1}}{\partial p_{\mathrm{R}}{}^2}$ is given by

$$\frac{\partial^2 \varepsilon_{\mathrm{DF},1}}{\partial p_{\mathrm{R}}{}^2} = \frac{\partial^2 \varepsilon_{\mathrm{DF},1}}{\partial \gamma_1{}^2}\left(\frac{\partial \gamma_1}{\partial p_{\mathrm{R}}}\right)^2 + \frac{\partial \varepsilon_{\mathrm{DF},1}}{\partial \gamma_1}\frac{\partial^2 \gamma_1}{\partial p_{\mathrm{R}}{}^2}$$

$$= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{w_{\mathrm{DF},1}^2}{2}\right)\left(\frac{\partial \gamma_1}{\partial p_{\mathrm{R}}}\right)^2\left(\underbrace{-\frac{\partial^2 w_{\mathrm{DF},1}}{\partial \gamma_1{}^2}}_{<0}+\underbrace{\frac{\partial w_{\mathrm{DF},1}}{\partial \gamma_1}}_{>0}\left(w_{\mathrm{DF},1}\frac{\partial w_{\mathrm{DF},1}}{\partial \gamma_1}-\frac{\frac{\partial^2 \gamma_1}{\partial p_{\mathrm{R}}^2}}{\left(\frac{\partial \gamma_1}{\partial p_{\mathrm{R}}}\right)^2}\right)\right). \tag{17}$$

Furthermore, combining the power consumption constraint $p_{\mathrm{S}} + p_{\mathrm{R}} = 2p_t$ with (2), we have $\gamma_1 = \frac{(2p_t - p_{\mathrm{R}})|h_1|^2}{p_{\mathrm{R}}|h_{\mathrm{RI}}|^2 + \sigma_1^2}$, from which we can obtain that

$$\frac{\frac{\partial^2 \gamma_1}{\partial p_{\mathrm{R}}^2}}{\left(\frac{\partial \gamma_1}{\partial p_{\mathrm{R}}}\right)^2} = \frac{2|h_{\mathrm{RI}}|^2(p_{\mathrm{R}}|h_{\mathrm{RI}}| + \sigma_1^2)}{|h_1|^2\sigma_1^2 + 2p_t|h_1|^2|h_{\mathrm{RI}}|^2} = \frac{2|h_{\mathrm{RI}}|^2}{|h_{\mathrm{RI}}|^2\gamma_1 + |h_1|^2} < \frac{2}{\gamma_1}. \tag{18}$$

On the other hand, based on (14), we have

$$\gamma_1\frac{\partial w_{\mathrm{DF},1}}{\partial \gamma_1} \geq \sqrt{\frac{M}{2}}\underbrace{\frac{\gamma_1\left(\gamma_1(\gamma_1 + 2) - \ln(1 + \gamma_1) - \frac{1}{e}\right)}{(\ln 2)^3 V(\gamma_1)^{\frac{3}{2}}(1 + \gamma_1)^3}}_{g_1(\gamma_1)}, \tag{19}$$

where $g_1(\gamma_1)$ is a function only in γ_1. Then, a monotonically increasing property of function $g_1(\gamma_1)$ in γ_1 can be easily proved when $\gamma_1 \geq 1$, from which we can obtain that $g_1(\gamma_1) \geq g_1(1) > 0.37$ holds $\forall \gamma_1 \geq 1$. The proof is provided in Appendix A. As a result, it holds that $\frac{\partial w_{\mathrm{DF},1}}{\partial \gamma_1} \geq \sqrt{\frac{M}{2}}\frac{0.37}{\gamma_1}$, $\forall \gamma_1 \geq 1$. Recall the condition (11), we have

$$w_{\mathrm{DF},1} = Q^{-1}(\varepsilon_{\mathrm{DF},1}) \geq Q^{-1}(\varepsilon_{\max}) \geq \frac{2\sqrt{2}}{0.37\sqrt{M_{\min}}}, \tag{20}$$

when $\varepsilon_{\mathrm{DF},1} \leq \varepsilon_{\max}$. Therefore, when $M \geq M_{\min}$ and $\gamma_1 \geq 1$, it holds that

$$w_{\mathrm{DF},1}\frac{\partial w_{\mathrm{DF},1}}{\partial \gamma_1} \geq \sqrt{\frac{M}{M_{\min}}}\frac{2}{\gamma_1} \geq \frac{2}{\gamma_1} > \frac{\frac{\partial^2 \gamma_1}{\partial p_{\mathrm{R}}^2}}{\left(\frac{\partial \gamma_1}{\partial p_{\mathrm{R}}}\right)^2}. \tag{21}$$

Then, by observing (17), we find that $\frac{\partial^2 \varepsilon_{\mathrm{DF},1}}{\partial p_{\mathrm{R}}{}^2} \geq 0$ holds if $M \geq M_{\min}$, $\gamma_1 \geq 1$ and $\varepsilon_{\mathrm{DF}} \leq \varepsilon_{\max}$. $\square$

Remark. *Note that in our model, considering practical systems, we have assumed the error probability $\varepsilon_{\mathrm{DF}} \leq \varepsilon_{\max} = 10^{-1}$ and the blocklength $M \geq M_{\min} = 200$. Since $\varepsilon_{\max} = 10^{-1}$ and $M_{\min} = 200$ satisfies the condition (11), the convexity of the objective function in problem (10) is also guaranteed under these intuitive assumptions. It should also be pointed out that the convexity also holds when we relax the constraint on blocklength M so that $\varepsilon_{\max} = 10^{-1}$ and $M_{\min} = 36$ (which is out of the assumption of this paper).*

According to Proposition 1, there exists a global optimal solution to Problem (10), which can be solved efficiently via convex optimization tools.

B. Achievable FBL Reliability of FD AF Relaying

According to (8), the overall error probability of the transmission via a DF AD relay is

$$\varepsilon_{\mathrm{AF}} = \mathcal{P}\left(\gamma_{\mathrm{AF},2}, \frac{D}{M-n}, M-n\right). \tag{22}$$

We consider to minimize the error probability by applying optimal power allocation

$$\begin{aligned}
\min_{p_{\mathrm{R}}} \quad & \varepsilon_{\mathrm{AF}} \\
s.t. \quad & p_{\mathrm{S}} = \tfrac{M}{M-n}p_t - p_{\mathrm{R}}, \\
& 0 < p_{\mathrm{R}} < \tfrac{M}{M-n}p_t,
\end{aligned} \tag{23}$$

where the first constraint is due to $(M-n)p_{\mathrm{S}} + (M-n)p_{\mathrm{R}} = Mp_t$. According to the proof of Proposition 1, by observing (12), it is clear that the error probability given in (8) is decreasing in the corresponding SNR/SINR. Hence, solving Problem (23) is the same as maximizing $\gamma_{\mathrm{AF},2}$, which is given by

$$\begin{aligned}
\max_{p_{\mathrm{R}}} \quad & \gamma_{\mathrm{AF},2} \\
s.t. \quad & p_{\mathrm{S}} = \tfrac{M}{M-n}p_t - p_{\mathrm{R}}, \\
& 0 < p_{\mathrm{R}} < \tfrac{M}{M-n}p_t.
\end{aligned} \tag{24}$$

Proposition 2. *Problem* (24) *is a pseudo-convex problem.*

Proof. By substituting $p_{\mathrm{S}} = \frac{M}{M-n}p_t - p_{\mathrm{R}}$ into (6), we have $\gamma_{\mathrm{AF},2} = \frac{A(p_{\mathrm{R}})}{B(p_{\mathrm{R}})}$ where

$$A(p_{\mathrm{R}}) = -p_{\mathrm{R}}{}^2 |h_1|^2 |h_2|^2 + p_{\mathrm{R}}\frac{M}{M-n}|h_1|^2|h_2|^2 p_t, \tag{25}$$

$$B(p_{\mathrm{R}}) = p_{\mathrm{R}}^2 |h_{\mathrm{RI}}|^2 |h_2|^2 + p_{\mathrm{R}}\left(|h_2|^2 \sigma_1^2 - |h_1|^2 \sigma_2^2 + |h_{\mathrm{RI}}|^2 \sigma_2^2\right) + \sigma_2^2\left(\frac{M}{M-n}p_t|h_1|^2 + \sigma_1^2\right). \tag{26}$$

Note that both $A(p_{\mathrm{R}})$ and $B(p_{\mathrm{R}})$ are quadratic functions with respect to p_{R}. It is easy to show that $A(p_{\mathrm{R}})$ is concave in p_{R} and $B(p_{\mathrm{R}})$ is convex in p_{R}, respectively. According to the results in Section 3.4.5 of [32], $\gamma_{\mathrm{AF},2}$ is pseudo-concave in p_{R} under the constraint $p_{\mathrm{S}} + p_{\mathrm{R}} = \frac{M}{M-n}p_t$. Hence, Problem (24) is pseudo-convex. □

According to the Proposition 2, the Problem (24) can be efficiently solved by Dinkelbach algorithm (in polynomial time) [33].

IV. MULTI-CARRIER SCENARIO: JOINT POWER AND DATA ALLOCATION FOR FBL RELIABILITY

In this section, we extend the study to a scenario with L parallel carriers, where the channel (of either the S-R link, the R-R self-interference link or the R-D) becomes a set of L channels with different instantaneous fading gains. In particular, we denote on the l-th carrier the gains of
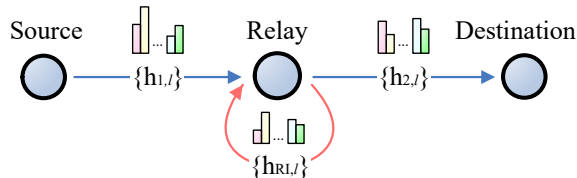
Fig. 3. Example of the considered two-hop FD relaying scenario with multiple carriers.

the S-R channel, the self-interference channel and the R-D channel by $|h_{1,l}|^2$, $|h_{\mathrm{RI},l}|^2$ and $|h_{2,l}|^2$, and let $p_{\mathrm{S},l}$ and $p_{\mathrm{R},l}$ represent the power allocated at the source and relay for the l-th carrier.

A. Achievable FBL Reliability of FD DF Relaying

Under the DF relaying scheme, the SNR of the l-th channel at the relay is given by $\gamma_{1,l} = \frac{p_{\mathrm{S},l}|h_{1,l}|^2}{p_{\mathrm{R},l}|h_{\mathrm{RI},l}|^2 + \sigma_{1,l}^2}$, $l \in \mathcal{L}$, where $\mathcal{L} \triangleq \{1, ..., L\}$. In addition, the SNR of the l-th channel in the second hop is given by $\gamma_{2,l} = \frac{p_{\mathrm{R},l}|h_{1,l}|^2}{\sigma_{2,l}^2}$, $l \in \mathcal{L}$.

In this subsection, we consider the case that the whole data packet with size D is separated into L sub-packets of size $D_{1,l}$, i.e., $\sum_{l=1}^{L} D_{1,l} = D$, and the DF relay and the destination is assumed to decode these sub-packets via different carriers in parallel[2]. In other words, the whole data packet with size D is successfully decoded, if and only if all the L sub-packets via different carriers are correctly decoded. Then, under the multi-carrier scenario the error probability at the DF relay is given by

$$\varepsilon_{\mathrm{Mc\text{-}DF},1} = 1 - \prod_{l=1}^{L}(1 - \varepsilon_{\mathrm{DF},1,l}) = 1 - \prod_{l=1}^{L}\left(1 - \mathcal{P}\left(\gamma_{1,l}, \frac{2D_{1,l}}{M}, \frac{M}{2}\right)\right). \tag{27}$$

Similarly, the decoded packet at the relay node is again splitted into L subpackets, which are then forwarded via the L carriers independently. Thus, the error probability at the destination is

$$\varepsilon_{\mathrm{Mc\text{-}DF},2} = 1 - \prod_{l=1}^{L}(1 - \varepsilon_{\mathrm{DF},2,l}) = 1 - \prod_{l=1}^{L}\left(1 - \mathcal{P}\left(\gamma_{2,l}, \frac{2D_{2,l}}{M}, \frac{M}{2}\right)\right), \tag{28}$$

where $D_{2,l}$ is the size of the sub-packet forward from the DF relay to the destination via the l-th carrier, i.e., $\sum_{l=1}^{L} D_{2,l} = D$.

According to (9) and the approximation applied in (9), the overall error probability of transmitting a data packet via the two-hop FD DF relaying can be obtained as

---

[2]From a practical point of view, most wireless communication standards such as LTE or 5G perform a resource block allocation in a way such that a time-frequency block is allocated for one frame. Moreover, the channel realization within this block is assumed to be flat-fading and characterized by a channel quality indicator (CQI) value, and the modulation and coding scheme (MCS) is then chosen accordingly. If multiple blocks with different CQIs are requested for one link, then multiple radio bearers are established in parallel and blocks transmitted over them are coded and decoded independently. Hence, although theoretically the packet can be jointly encoded and decoded among all the carriers (with more likely different channel realizations), we in this work refer to a more practical assumption of encoding and decoding in parallel. On the other hand, according to [34]–[36], joint decoding (over multiple carriers) also requires a significantly high complexity. Therefore, the separate encoding for different carriers desiring a relatively lower complexity is more favorable in the considered latency-constrained scenarios.

$$\varepsilon_{\text{Mc-DF}} = \varepsilon_{\text{Mc-DF},1} + \varepsilon_{\text{Mc-DF},2} - \varepsilon_{\text{Mc-DF},1}\varepsilon_{\text{Mc-DF},2}$$

$$\approx \sum_{l=1}^{L}\sum_{i=1}^{2}\varepsilon_{\text{DF},i,l} = \sum_{l=1}^{L}\left[\mathcal{P}\left(\gamma_{1,l}, \frac{2D_{1,l}}{M}, \frac{M}{2}\right) + \mathcal{P}\left(\gamma_{2,l}, \frac{2D_{2,l}}{M}, \frac{M}{2}\right)\right]. \tag{29}$$

Clearly, $\varepsilon_{\text{Mc-DF}}$ is a polynomial of $\varepsilon_{\text{DF},i,l}$. The approximation in (29) is introduced by ignoring all the higher terms of the polynomial and keeps only the first-order terms, which are tight in a reliable transmission scenario, i.e., $\varepsilon_{\text{DF},i,l} \leq 10^{-1}$ and $\varepsilon_{\text{DF},i,l} \gg \varepsilon_{\text{DF},i,l} \cdot \varepsilon_{\text{DF},i,l'}, \forall l, l' \in \mathcal{L}$.

We consider to minimize the overall error probability by applying optimal joint power and data allocation among multi-carriers in the two hops of FD DF relaying, which is formulated as

$$\min_{\{p_{\text{S},l}, p_{\text{R},l}, D_{1,l}, D_{2,l}\}} \quad \sum_{l=1}^{L}\sum_{i=1}^{2}\varepsilon_{\text{DF},i,l}$$

$$s.t. \quad \sum_{l=1}^{L}\left(p_{\text{S},l} + p_{\text{R},l}\right) \leq 2p_t,$$

$$\sum_{l=1}^{L}D_{1,l} = D, \quad \sum_{l=1}^{L}D_{2,l} = D, \tag{30}$$

$$p_{\text{S},l} > 0, \quad p_{\text{R},l} > 0, \quad \forall l \in \mathcal{L},$$

$$D_{i,l} \geq 0, \quad \forall i \in \{1,2\}, \quad \forall l \in \mathcal{L}.$$

In particular, the power allocation are among all $2L$ carriers of the two hops, while the data allocation is performed per hop, i.e., all the data should be transmitted via each of the hops.

However, the objective of Problem (30) cannot be proved to be jointly convex in $p_{\text{S},l}$, $p_{\text{R},l}$, $D_{1,l}$ and $D_{2,l}$. Further notice that the allocated power $p_{\text{S},l}$ and $p_{\text{R},l}$ can be uniquely determined by SNRs $\gamma_{1,l}$ and $\gamma_{2,l}$, i.e., $\forall l \in \mathcal{L}$,

$$p_{\text{S},l}(\gamma_{1,l}, \gamma_{2,l}) = \frac{|h_{\text{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\gamma_{1,l}\gamma_{2,l} + \frac{\sigma_{1,l}^2}{|h_{1,l}|^2}\gamma_{1,l}, \tag{31}$$

$$p_{\text{R},l}(\gamma_{2,l}) = \frac{\sigma_{2,l}^2}{|h_{2,l}|^2}\gamma_{2,l}. \tag{32}$$

Thus, by substituting variables $p_{\text{S},l}$ and $p_{\text{R},l}$ with $\gamma_{1,l}$ and $\gamma_{2,l}$, the Problem (30) can be equivalently reformulated to

$$\min_{\{\gamma_{1,l}, \gamma_{2,l}, D_{1,l}, D_{2,l}\}} \quad \sum_{l=1}^{L}\sum_{i=1}^{2}\varepsilon_{\text{DF},i,l}$$

$$s.t. \quad \sum_{l=1}^{L}\left(p_{\text{S},l}(\gamma_{1,l}, \gamma_{2,l}) + p_{\text{R},l}(\gamma_{2,l})\right) \leq 2p_t,$$

$$\sum_{l=1}^{L}D_{1,l} = D, \quad \sum_{l=1}^{L}D_{2,l} = D, \tag{33}$$

$$\gamma_{i,l} \geq 1, \quad D_{i,l} \geq 0, \quad \forall i \in \{1,2\}, \quad \forall l \in \mathcal{L}.$$

Note that the constraints $\gamma_{i,l} \geq 1$ in Problem (33) result from the assumption of reliable

transmission. In other words, although the constraints of $p_{\mathrm{S},l} > 0$ and $p_{\mathrm{R},l} > 0$ in Problem (30) are not tightly mathematically equivalent to the constraints $\gamma_{i,l} \geq 1$ in Problem (33), these two problems are still equivalent in reliable transmission scenario. Then, for the Problem (33), we have the following Proposition 3.

Proposition 3. *Under the assumptions that the error probability $\varepsilon_{\mathrm{DF},i,l} \leq \varepsilon_{\max}$, the blocklength $M \geq M_{\min}$ and the SNR $\gamma_{i,l} \geq 1$, $\forall i \in \{1, 2\}$, $\forall l \in \mathcal{L}$, the objective of Problem (33) is convex when the following inequalities regarding $\varepsilon_{\max}$ and $M_{\min}$ hold:*

$$0 < \frac{\ln(M_{\min}/2)}{M_{\min}/2} < 0.9 - \ln 2; \tag{34}$$

$$\varepsilon_{\max} \leq Q\left(\sqrt{\frac{3}{2M_{\min}}} \frac{1}{9 - 10(\ln 2 + \frac{\ln(M_{\min}/2)}{M_{\min}/2})}\right). \tag{35}$$

Proof. We complete the proof by showing the convexity of $\varepsilon_{\mathrm{DF},i,l}$ with respect to $(\gamma_{i,l}, D_{i,l})$. The Hessian matrix of $\varepsilon_{\mathrm{DF},i,l}$ to $(\gamma_{i,l}, D_{i,l})$ is expressed as

$$\mathbf{H} = \begin{pmatrix} \frac{\partial^2 \varepsilon_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}^2} & \frac{\partial^2 \varepsilon_{\mathrm{DF},i,l}}{\partial \gamma_{i,l} \partial D_{i,l}} \\ \frac{\partial^2 \varepsilon_{\mathrm{DF},i,l}}{\partial D_{i,l} \partial \gamma_{i,l}} & \frac{\partial^2 \varepsilon_{\mathrm{DF},i,l}}{\partial D_{i,l}^2} \end{pmatrix} = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{w_{\mathrm{DF},i,l}^2}{2}\right) \mathbf{H}', \tag{36}$$

where $\mathbf{H}'$ is given by

$$\mathbf{H}' = \begin{pmatrix} w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}}\right)^2 - \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}^2} & w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}} \frac{\partial w_{\mathrm{DF},i,l}}{\partial D_{i,l}}\right) - \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l} \partial D_{i,l}} \\ w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}} \frac{\partial w_{\mathrm{DF},i,l}}{\partial D_{i,l}}\right) - \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l} \partial D_{i,l}} & w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial D_{i,l}}\right)^2 - \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial D_{i,l}^2} \end{pmatrix}, \tag{37}$$

with $w_{\mathrm{DF},i,l} = \frac{\mathcal{C}(\gamma_{i,l}) + \frac{\log_2(M/2)}{M/2} - 2D_{i,l}/M}{\sqrt{2V(\gamma_{i,l})/M}}$.

According to (13), we have $\frac{\partial^2 \varepsilon_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}^2} \geq 0$. In addition, we have the determinant $|\mathbf{H}'|$ as

$$|\mathbf{H}'| = \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l} \partial D_{i,l}} \left[2w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}} \frac{\partial w_{\mathrm{DF},i,l}}{\partial D_{i,l}}\right) - \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l} \partial D_{i,l}}\right] - w_{\mathrm{DF},i,l}\left(\frac{\partial w_{\mathrm{DF},i,l}}{\partial D_{i,l}}\right)^2 \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}^2}$$

$$= \frac{\sqrt{2/M}}{(\ln 2)^2 V(\gamma_{i,l})^{\frac{3}{2}}(1+\gamma_{i,l})^3}\left[-\frac{4w_{\mathrm{DF},i,l}}{\sqrt{2MV(\gamma_{i,l})}} \frac{\partial w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}} - \frac{\sqrt{2/M}}{(\ln 2)^2 V(\gamma_{i,l})^{\frac{3}{2}}(1+\gamma_{i,l})^3}\right] - \frac{2w_{\mathrm{DF},i,l}}{MV(\gamma_{i,l})} \frac{\partial^2 w_{\mathrm{DF},i,l}}{\partial \gamma_{i,l}^2}$$

$$= \frac{w_{\mathrm{DF},i,l}\sqrt{2/M}}{(\ln 2)^5 V(\gamma_{i,l})^{\frac{7}{2}}(1+\gamma_{i,l})^6}\left[\left((1+\gamma_{i,l})^2 - 1\right)^2 - \frac{\sqrt{2/M}}{w_{\mathrm{DF},i,l}}\sqrt{1 - \frac{1}{(1+\gamma_{i,l})^2}}\right. \tag{38}$$

$$\left. + (2 - 3(1+\gamma_{i,l})^2)\left(\ln(1+\gamma_{i,l}) + \frac{\ln(M/2)}{M/2} - \frac{2D\ln 2}{M}\right)\right]$$

$$> \frac{w_{\mathrm{DF},i,l}\sqrt{2/M}}{(\ln 2)^5 V(\gamma_{i,l})^{\frac{7}{2}}(1+\gamma_{i,l})^6} g_2(\gamma_{i,l}).$$

where the function $g_2(\gamma_{i,l})$ is given by

$$g_2(\gamma_{i,l}) = \left((1+\gamma_{i,l})^2-1\right)^2 + \left(2-3\left(1+\gamma_{i,l}\right)^2\right)\left(\ln(1+\gamma_{i,l})+\frac{\ln(M_{\min}/2)}{M_{\min}/2}\right) - \frac{\sqrt{2/M_{\min}}}{Q^{-1}(\varepsilon_{\max})}\sqrt{1-\frac{1}{(1+\gamma_{i,l})^2}}$$

(39)

The inequality in (38) hods due to the facts that $\frac{\ln(M/2)}{M/2} - \frac{2D\ln 2}{M} \leq \frac{\ln(M/2)}{M/2} \leq \frac{\ln(M_{\min}/2)}{M_{\min}/2}$ when (34) holds, $2-3(1+\gamma_{i,l})^2 < 0$ when $\gamma_{i,l} \geq 1$ and $w_{\mathrm{DF},i,l} = Q^{-1}(\varepsilon_{\mathrm{DF},i,l}) \geq Q^{-1}(\varepsilon_{\max})$. Moreover, $g_2(\gamma_{i,l})$ is a pure function of $\gamma_{i,l}$ and a monotonically increasing property can be easily proved when $\gamma_{i,l} \geq 1$, while the proof is provided in Appendix B. Namely, we have $g_2(\gamma_{i,l}) \geq g_2(1)$ when $\gamma_{i,l} \geq 1$. Under the condition (35) between $\varepsilon_{\max}$ and $M_{\min}$, we can easily obtain that $g_2(1) \geq 0$, so that $g_2(\gamma_{i,l}) \geq 0$ holds $\forall \gamma_{i,l} \geq 1$. As a result, $|\mathbf{H}| = \frac{1}{\sqrt{2\pi}}\exp\left(-w_{\mathrm{DF},i,l}^2\right)|\mathbf{H'}| > 0$. Therefore, $\mathbf{H}$ is positive definite in $(\gamma_{i,l}, D_{i,l})$. Accordingly, the objective of Problem (33), which is the accumulation of error probability $\varepsilon_{\mathrm{DF},i,l}$, is also convex in $(\gamma_{i,l}, D_{i,l})$. $\square$

Remark. *Based on the proposition above, we can have more intuitive assumptions that $\varepsilon_{\max} = 10^{-1}$ and $M_{\min} = 200$ satisfying the conditions (34) and (35), so that in our model the convexity of the objective in Problem (33) holds according to the Proposition 3. In fact, while guaranteeing the convexity, the constraint on blocklength M can also be relaxed to $M \geq M_{\min} = 36$ (which is much looser than the interested blocklength region in this paper).*

However, Problem (33) is still not convex due to the non-convexity of the first constraint. To address the non-convexity, we next construct a tight convex approximation of the first constraint in Problem (33) and subsequently propose an iterative algorithm for the solution.

1) Convex approximation: Note that the Problem (33) can be approximated to a convex one, only when we find a tight convex approximation of function $p_{\mathrm{S},l}(\gamma_{1,l}, \gamma_{2,l})$, i.e., with given local point $(\gamma_{1,l}^{(r)}, \gamma_{2,l}^{(r)})$ constructing a convex function $p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l})$ so that $p_{\mathrm{S},l}(\gamma_{1,l}, \gamma_{2,l}) \leq p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l})$ for any feasible point $(\gamma_{1,l}, \gamma_{2,l})$ and the equality holds when $(\gamma_{1,l}, \gamma_{2,l}) = (\gamma_{1,l}^{(r)}, \gamma_{2,l}^{(r)})$. Based on (31) and the inequality that $xy = \frac{1}{F}Fxy \leq \frac{F}{2}x^2 + \frac{1}{2F}y^2, \forall x, y, F > 0$, in which the equality holds when $Fx = y$, we have

$$p_{\mathrm{S},l}(\gamma_{1,l}, \gamma_{2,l}) = \frac{|h_{\mathrm{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\cdot\underline{\underline{\gamma_{1,l}}}\cdot\underline{\underline{\gamma_{2,l}}} + \frac{\sigma_{1,l}^2}{|h_{1,l}|^2}\gamma_{1,l} \leq \frac{|h_{\mathrm{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\left(\frac{F_l^{(r)}\gamma_{1,l}^2}{\underline{\underline{2}}} + \frac{\gamma_{2,l}^2}{\underline{\underline{2F_l^{(r)}}}}\right) + \frac{\sigma_{1,l}^2}{|h_{1,l}|^2}\gamma_{1,l}$$

$$\overset{\Delta}{=} p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l}), \tag{40}$$

where $F_l^{(r)}$ is defined as a positive constant

$$F_l^{(r)} = \frac{\gamma_{2,l}^{(r)}}{\gamma_{1,l}^{(r)}}. \tag{41}$$

Clearly, the constructed $p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l})$ in (40) is a convex function and the equality in (40) holds

when $(\gamma_{1,l}, \gamma_{2,l}) = (\gamma_{1,l}^{(r)}, \gamma_{2,l}^{(r)})$. By replacing $p_{\mathrm{S},l}(\gamma_{1,l}, \gamma_{2,l})$ with its approximation $p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l})$, the Problem (33) can be approximated as

$$
\begin{aligned}
\min_{\{\gamma_{1,l}, \gamma_{2,l}, D_{1,l}, D_{2,l}\}} \quad & \sum_{l=1}^{L} \sum_{i=1}^{2} \varepsilon_{\mathrm{DF},i,l} \\
s.t. \quad & \sum_{l=1}^{L} \left(p_{\mathrm{S},l}^{(r)}(\gamma_{1,l}, \gamma_{2,l}) + p_{\mathrm{R},l}(\gamma_{2,l}) \right) \leq 2p_t, \\
& \sum_{l=1}^{L} D_{1,l} = D, \quad \sum_{l=1}^{L} D_{2,l} = D, \\
& \gamma_{i,l} \geq 1, \quad D_{i,l} \geq 0, \quad \forall i \in \{1, 2\}, \quad \forall l \in \mathcal{L}.
\end{aligned}
\tag{42}
$$

The Problem (42) is clearly a convex problem and it should be mentioned that due to the approximation (40), the feasible set of Problem (42) is a subset of the feasible set in Problem (33).

2) Iterative solution: With the assistance of constructed convex approximation, we solve the Problem (33) in an iterative manner. In the initialization step, we set $r = 0$ and start with a feasible point $(\gamma_{i,l}^{(0)}, D_{i,l}^{(0)})$ with $i \in \{1, 2\}$ and $l \in \mathcal{L}$.

In the r-th iteration, we construct the corresponding convex Problem (42) at the local point $(\gamma_{i,l}^{(r)}, D_{i,l}^{(r)})$. Through convex optimization tools, the optimal solution for Problem (42) can be obtained as $(\gamma_{i,l}^{(r\star)}, D_{i,l}^{(r\star)})$, which will be applied as the local point in the next iteration, i.e., $(\gamma_{i,l}^{(r+1)}, D_{i,l}^{(r+1)}) = (\gamma_{i,l}^{(r\star)}, D_{i,l}^{(r\star)})$. Note that in each iteration, we have

$$
\sum_{l=1}^{L} \sum_{i=1}^{2} \varepsilon_{\mathrm{DF},i,l}(\gamma_{i,l}^{(r+1)}, D_{i,l}^{(r+1)}) = \sum_{l=1}^{L} \sum_{i=1}^{2} \varepsilon_{\mathrm{DF},i,l}(\gamma_{i,l}^{(r\star)}, D_{i,l}^{(r\star)}) \leq \sum_{l=1}^{L} \sum_{i=1}^{2} \varepsilon_{\mathrm{DF},i,l}(\gamma_{i,l}^{(r)}, D_{i,l}^{(r)}). \tag{43}
$$

Therefore, by repeating the iterations, the overall error probability will continuously decrease and eventually converge to a sub-optimal point. The algorithm flow is shown in Algorithm 1.

Algorithm 1 : Iterative Algorithm for Multi-Carrier FD DF Relaying.

---

a) Initialize a local point $(\gamma_{i,l}^{(0)}, D_{i,l}^{(0)})$ for Problem (33).
b) Choose a threshold $\lambda_{th} \geq 0$ and set $r = 0$.
c) Construct Problem (42) based on local point $(\gamma_{i,l}^{(r)}, D_{i,l}^{(r)})$.
d) Solve Problem (42) and get optimal point $(\gamma_{i,l}^{(r\star)}, D_{i,l}^{(r\star)})$.
e) If the reduction of error probability is larger than λ_{th},
  $(\gamma_{i,l}^{(r+1)}, D_{i,l}^{(r+1)}) = (\gamma_{i,l}^{(r\star)}, D_{i,l}^{(r\star)})$.
  $r = r + 1$, **Back to c)**.
  End
f) According to (31) and (32), calculate $p_{\mathrm{S},l}$ and $p_{\mathrm{R},l}$ from obtained $\gamma_{i,l}$.

Hereby, in concern of practical deployment, we provide a complexity analysis for the proposed algorithm. In the proposed algorithm, each iteration contains a convex optimization task over

in total $4L$ variables. Following the complexity analysis strategy in [37], based on the ellipsoid method [38], the computational complexity of the proposed algorithm for FD DF relaying in multi-carrier scenario is given by $\mathcal{O}(\varphi(4L)^4)$, where φ denotes the number of iterations. Clearly, with more carriers assigned for resource allocation, the complexity will increase in a large degree.

B. Achievable FBL Reliability of FD AF Relaying

For AF relaying, the resulting SNR at the destination of the l-th channel is denoted by

$$\gamma_{\mathrm{AF},l} = \frac{G_l p_{\mathrm{S},l} p_{\mathrm{R},l} |h_{1,l}|^2 |h_{2,l}|^2}{G_l p_{\mathrm{R},l}^2 |h_{\mathrm{RI},l}|^2 |h_{2,l}|^2 + G_l p_{\mathrm{R},l} |h_{2,l}|^2 \sigma_{1,l}^2 + \sigma_{2,l}^2}, \tag{44}$$

where $G_l = \frac{1}{p_{\mathrm{S},l}|h_{1,l}|^2 + p_{\mathrm{R},l}|h_{\mathrm{RI},l}|^2 + \sigma_{1,l}^2}$ is the gain of the amplifier for the l-th carrier. We assume that the whole packet with size D is split into L sub-packets of size D_l, $l \in \mathcal{L}$, i.e., $\sum_{l=1}^{L} D_l = D$, and these sub-packets are transmitted through different carriers. Different from DF relaying scheme, the data packet with size D is only split once, i.e., at the source node, since there is no decoding behaviour at the relay so that resplitting the data packet is impossible. Similarly, the destination decodes L sub-packets independently. Therefore, the data packet is correctly decoded only when all sub-packets are accurately decoded. Then, the overall error probability under AF relaying scheme is given by

$$\varepsilon_{\mathrm{Mc\text{-}AF}} = 1 - \prod_{l=1}^{L} (1 - \varepsilon_{\mathrm{AF},l}) = 1 - \prod_{l=1}^{L} \left(1 - \mathcal{P}\left(\gamma_{\mathrm{AF},l}, \frac{D_l}{M-n}, M-n \right) \right). \tag{45}$$

By ignoring the higher-order terms, which are neglectable under a reliable transmission, i.e., $\varepsilon_{\mathrm{AF},l}\varepsilon_{\mathrm{AF},l'} \ll \varepsilon_{\mathrm{AF},l}$ $\forall l, l' \in \mathcal{L}$ when $\varepsilon_{\mathrm{AF},l} < 0.1$, the error probability in (45) can be tightly approximated as

$$\varepsilon_{\mathrm{Mc\text{-}AF}} \approx \sum_{l=1}^{L} \varepsilon_{\mathrm{AF},l} = \sum_{l=1}^{L} \mathcal{P}\left(\gamma_{\mathrm{AF},l}, \frac{D_l}{M-n}, M-n \right). \tag{46}$$

Then, aiming at minimizing the overall error probability, the problem is formulated as

$$\begin{aligned}
\min_{\{p_{\mathrm{S},l}, p_{\mathrm{R},l}, D_l\}} \quad & \sum_{l=1}^{L} \varepsilon_{\mathrm{AF},l} \\
\mathrm{s.t.} \quad & \sum_{l=1}^{L} (p_{\mathrm{S},l} + p_{\mathrm{R},l}) \leq \frac{M}{M-n} p_t, \\
& \sum_{l=1}^{L} D_l = D, \\
& p_{\mathrm{S},l} > 0, \ p_{\mathrm{R},l} > 0, \ D_l \geq 0, \ \forall l \in \mathcal{L}.
\end{aligned} \tag{47}$$

Note that the first constraint in Problem (47) corresponds to the power constraint

$$\sum_{l=1}^{L} ((M-n)p_{\mathrm{S},l} + (M-n)p_{\mathrm{R},l}) \leq M p_t. \tag{48}$$

Since the convexity of the objective in Problem (47) cannot be proved, we similarly turn to treat the SNRs $\gamma_{\text{AF},l}$ as the variables to be optimized. According to (44), we have

$$p_{\text{S},l} = \frac{\gamma_{\text{AF},l}\left(p_{\text{R},l}^2|h_{\text{RI},l}|^2|h_{2,l}|^2 + p_{\text{R},l}|h_{2,l}|^2\sigma_{1,l}^2 + p_{\text{R},l}|h_{\text{RI},l}|^2\sigma_{2,l}^2 + \sigma_{1,l}^2\sigma_{2,l}^2\right)}{(p_{\text{R},l}|h_{2,l}|^2 - \gamma_{\text{AF},l}\sigma_{2,l}^2)|h_{1,l}|^2}. \tag{49}$$

Clearly, the power variables $p_{\text{S},l}$ and $p_{\text{R},l}$ cannot be completely substituted with only SNR variables $\gamma_{\text{AF},l}$. Therefore, for the problem reformulation, we introduce auxiliary variables a_l, $l \in \mathcal{L}$, which are defined as

$$a_l = p_{\text{R},l}|h_{2,l}|^2 - \gamma_{\text{AF},l}\sigma_{2,l}^2. \tag{50}$$

Namely, we have

$$p_{\text{R},l}(a_l, \gamma_{\text{AF},l}) = \frac{1}{|h_{2,l}|^2}(a_l + \gamma_{\text{AF},l}\sigma_{2,l}^2). \tag{51}$$

According to (49), $a_l > 0$ holds so that a non-negative $p_{\text{S},l}$ can be guaranteed.

As a result, the allocated power at source can be denoted as

$$p_{\text{S},l}(a_l, \gamma_{\text{AF},l})$$

$$= \frac{\gamma_{\text{AF},l}}{a_l|h_{1,l}|^2}\left[(p_{\text{R},l}(a_l, \gamma_{\text{AF},l}))^2|h_{\text{RI},l}|^2|h_{2,l}|^2 + p_{\text{R},l}(a_l, \gamma_{\text{AF},l})\left(|h_{2,l}|^2\sigma_{1,l}^2 + |h_{\text{RI},l}|^2\sigma_{2,l}^2\right) + \sigma_{1,l}^2\sigma_{2,l}^2\right] \tag{52}$$

$$= a_l f_{1,l}(\gamma_{\text{AF},l}) + \frac{1}{a_l}f_{2,l}(\gamma_{\text{AF},l}) + f_{3,l}(\gamma_{\text{AF},l}),$$

where the functions $f_{j,l}(\gamma_{\text{AF},l})$, $j \in \{1, 2, 3\}$ are all polynomials of $\gamma_{\text{AF},l}$, which are given by

$$f_{1,l}(\gamma_{\text{AF},l}) = \frac{|h_{\text{RI},l}|^2}{|h_{1,l}|^2|h_{2,l}|^2}\gamma_{\text{AF},l}, \tag{53}$$

$$f_{2,l}(\gamma_{\text{AF},l}) = \frac{|h_{\text{RI},l}|^2\sigma_{2,l}^4}{|h_{1,l}|^2|h_{2,l}|^2}\gamma_{\text{AF},l}^3 + \frac{\sigma_{1,l}^2\sigma_{2,l}^2}{|h_{1,l}|^2}\gamma_{\text{AF},l} + \frac{|h_{2,l}|^2\sigma_{1,l}^2 + |h_{\text{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\sigma_{2,l}^2\gamma_{\text{AF},l}^2, \tag{54}$$

$$f_{3,l}(\gamma_{\text{AF},l}) = \frac{2|h_{\text{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\gamma_{\text{AF},l}^2 + \frac{|h_{2,l}|^2\sigma_{1,l}^2 + |h_{\text{RI},l}|^2\sigma_{2,l}^2}{|h_{1,l}|^2|h_{2,l}|^2}\gamma_{\text{AF},l}. \tag{55}$$

Then, by focusing on variables a_l, $\gamma_{\text{AF},l}$ and D_l, the Problem (47) can be reformulated to

$$\min_{\{a_l, \gamma_{\text{AF},l}, D_l\}} \sum_{l=1}^{L}\varepsilon_{\text{AF},l}$$

$$\text{s.t.} \quad \sum_{l=1}^{L}(p_{\text{S},l}(a_l, \gamma_{\text{AF},l}) + p_{\text{R},l}(a_l, \gamma_{\text{AF},l})) \leq \frac{M}{M-n}p_t, \tag{56}$$

$$\sum_{l=1}^{L}D_l = D,$$

$$a_l > 0, \ \gamma_{\text{AF},l} \geq 1, \ D_l \geq 0, \ \forall l \in \mathcal{L}.$$

Proposition 4. *Assuming a reliable transmission, i.e., error probability $\varepsilon_{\text{AF},l} < 10^{-1}$ and the SNR $\gamma_{\text{AF},l} \geq 1$, $\forall l \in \mathcal{L}$, the objective of Problem (56) is convex in $(\gamma_{\text{AF},l}, D_l)$ when the blocklength $M - n \geq 50$.*

Proof. By following the same procedure as in the proof of Proposition 3 and replacing $\frac{M}{2}$ with $M-n$, the convexity of the objective in Problem (56) can be easily proved when $M-n \geq 50$. $\square$

Note that when the blocklength $M \geq 200$, $M - n \geq 50$ can be easily satisfied since $n \ll \frac{M}{2}$ in general AF relaying scheme. If the amplification delay $n > \frac{M}{2}$, FD AF relaying is clearly not preferred in comparison to the DF one.

However, due to the non-convexity of $p_{\mathrm{S},l}(a_l, \gamma_{\mathrm{AF},l})$, the Problem (56) is still not convex. Similarly, we will propose a convex approximation and an iterative algorithm for iteratively addressing the problem.

1) Convex approximation: Similar to the strategy in DF relaying with multiple carriers, for an iterative solution, we are supposed to construct a tight convex approximation $p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l})$ for function $p_{\mathrm{S},l}(a_l, \gamma_{\mathrm{AF},l})$, so that for any feasible point we have $p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l}) \geq p_{\mathrm{S},l}(a_l, \gamma_{\mathrm{AF},l})$, where the equality holds at a point $(a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)})$. Based on (52) and the inequality that $xy = \frac{1}{F}Fxy \leq \frac{F}{2}x^2 + \frac{1}{2F}y^2$, $\forall x, y, F > 0$, with given local point $(a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)})$, we can obtain that

$$p_{\mathrm{S},l}(a_l, \gamma_{\mathrm{AF},l}) = \underline{a_l} \cdot \underline{f_{1,l}(\gamma_{\mathrm{AF},l})} + \underline{\frac{1}{a_l}} \cdot \underline{\underline{f_{2,l}(\gamma_{\mathrm{AF},l})}} + f_{3,l}(\gamma_{\mathrm{AF},l})$$

$$\leq \underline{\frac{F_{1,l}^{(r)}a_l^2}{2}} + \underline{\frac{(f_{1,l}(\gamma_{\mathrm{AF},l}))^2}{2F_{1,l}^{(r)}}} + \underline{\underline{\frac{F_{2,l}^{(r)}}{2a_l^2}}} + \underline{\underline{\frac{(f_{2,l}(\gamma_{\mathrm{AF},l}))^2}{2F_{2,l}^{(r)}}}} + f_{3,l}(\gamma_{\mathrm{AF},l}) \qquad (57)$$

$$\triangleq p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l}),$$

where $F_{1,l}^{(r)}$ and $F_{2,l}^{(r)}$ are positive constants defined by

$$F_{1,l}^{(r)} = \frac{f_{1,l}(\gamma_{\mathrm{AF},l}^{(r)})}{a_l^{(r)}}, \qquad (58)$$

$$F_{2,l}^{(r)} = a_l^{(r)} f_{2,l}(\gamma_{\mathrm{AF},l}^{(r)}). \qquad (59)$$

As the functions $f_{1,l}(\gamma_{\mathrm{AF},l})$, $f_{2,l}(\gamma_{\mathrm{AF},l})$ and $f_{3,l}(\gamma_{\mathrm{AF},l})$, defined in (53), (54) and (55), are all positive convex increasing functions when $\gamma_{\mathrm{AF},l} \geq 1$, the approximated function $p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l})$ is definitely convex. In addition, $p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l}) = p_{\mathrm{S},l}(a_l, \gamma_{\mathrm{AF},l})$ holds when $(a_l, \gamma_{\mathrm{AF},l}) = (a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)})$.

With the introduction of $p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l})$, the convex approximation of Problem (56) is given by

$$\begin{aligned} \min_{\{a_l, \gamma_{\mathrm{AF},l}, D_l\}} \quad & \sum_{l=1}^{L} \varepsilon_{\mathrm{AF},l} \\ s.t. \quad & \sum_{l=1}^{L} \left(p_{\mathrm{S},l}^{(r)}(a_l, \gamma_{\mathrm{AF},l}) + p_{\mathrm{R},l}(a_l, \gamma_{\mathrm{AF},l}) \right) \leq \frac{M}{M-n} p_t, \\ & \sum_{l=1}^{L} D_l = D, \\ & a_l > 0, \ \gamma_{\mathrm{AF},l} \geq 1, \ D_l \geq 0, \ \forall l \in \mathcal{L}. \end{aligned} \qquad (60)$$

Note that all feasible points in Problem (60) are also feasible in Problem (56), due to the tight approximation (57).

2) Iterative algorithm: Then, after the convex approximation, the Problem (56) can be iteratively addressed. After initialization, we have the initial local feasible point $(a_l^{(0)}, \gamma_{\mathrm{AF},l}^{(0)}, D_l^{(0)})$ and the initial iteration index $r = 0$.

In r-th iteration, we construct the convex approximation (60) on local point $(a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)}, D_l^{(r)})$. After optimally solving Problem (60), the optimal is directly applied as the local point for $r+1$-th iteration. While updating the local point, the objective, i.e., the overall error probability can be simply proved to be reduced. Thus, by repeating the iterations, the overall error probability will iteratively decrease and finally converge to a sub-optimal point. The algorithm flow is displayed in Algorithm 2. Following the same approach as in FD DF relaying with multiple carriers, we can obtain the complexity of the proposed algorithm for FD AF relaying in multi-carrier scenario. Since in each iteration the convex problem has in total $3L$ variables, the overall complexity can be represented as $\mathcal{O}(\varphi(3L)^4)$ with φ denoting the iteration number. Similarly, more carriers will lead to a larger computational complexity.

Algorithm 2 : Iterative Algorithm for Multi-Carrier FD AF Relaying.

---

a) Initialize a local point $(a_l^{(0)}, \gamma_{\mathrm{AF},l}^{(0)}, D_l^{(0)})$ for Problem (56).
b) Choose a threshold $\lambda_{th} \geq 0$ and set $r = 0$.
c) Construct Problem (60) on local point $(a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)}, D_l^{(r)})$.
d) Solve convex Problem (60).
e) **If** the reduction of error probability is larger than λ_{th},
    $r = r + 1$ and update$(a_l^{(r)}, \gamma_{\mathrm{AF},l}^{(r)}, D_l^{(r)})$.
    Back to c).
  End
f) Based on obtained $\gamma_{\mathrm{AF},l}$ and a_l, calculate allocated power $p_{\mathrm{S},l}$ and $p_{\mathrm{R},l}$ according to (51) and (52).

V. NUMERICAL RESULTS

In this section, via simulations, we validate our proposed resource allocation algorithms in FD relaying and evaluate the overall system performance. We consider the following default simulation setups in our numerical evaluations: the maximum average transmit power $p_t = 2\mathrm{W}$, noise power level $\sigma^2 = -90\mathrm{dBm}$, the blocklength $M = 200$ symbols, carrier number $L = 5$, the data packet size $D = 1.1ML = 1100$ bits and residual loop interference gain $|h_{\mathrm{RI}}|^2 = -108dB$. As for the channel gains, we adopt a general pathloss model from [39], in which the pathloss with
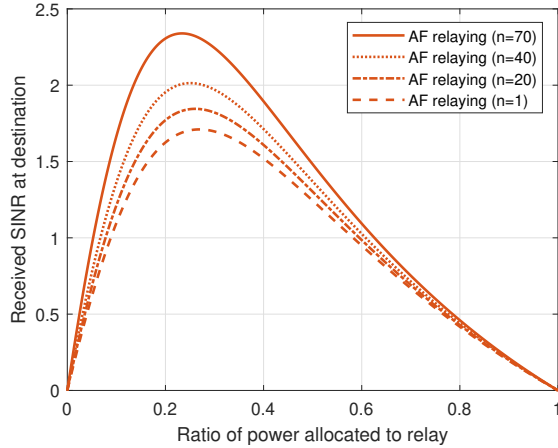
Fig. 4. The received SINR of FD AF relaying with single carrier under different power allocation ratio.
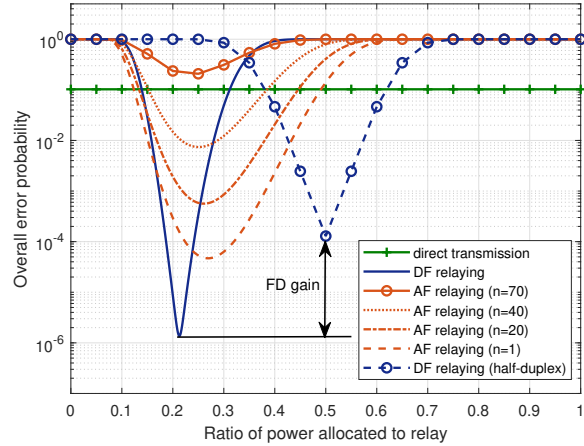
Fig. 5. Overall error probability comparison with single carrier.

a distance of d_i is obtained by $L_i = L(d_0) + 10\alpha \log(\frac{d_i}{d_0})$. The baseline pathloss $L(d_0)$ is defined based on a free-space pathloss model with a reference distance $d_0 = 100$m. In simulations, different carriers are assigned with different baseline pathloss according to the different carrier frequencies. We consider a linear topology and assume that the relay is installed between the source node and the destination node. The distances from relay to the source and from relay to the destination are denoted by d_1 and d_2, respectively, while $d_1 = d_2 = 250$m, and the direct distance from the source node to the destination node is then given by $d_1 + d_2 = 500$m. We apply a pathloss exponent $\alpha = 3.5$. Then the channel gain for each carrier on each link can be calculated correspondingly. In addition, as a benchmark, we also perform simulations on the direct transmission for a comparison with relaying schemes. The direct channel gain h_{direct} is determined based on the same pathloss model with a transmission distance of 500m. With the same setups, namely the same data packet size and the same blocklength, the error probability for the direct transmission is given by $\varepsilon_{\text{direct}} = \mathcal{P}\left(\gamma_{\text{direct}}, \frac{D}{M}, M\right)$, where $\gamma_{\text{direct}} = \frac{p_t|h_{\text{direct}}|^2}{\sigma_2^2}$ denotes the corresponding SNR.

In the following, we investigate the network performance with single and multiple carriers, respectively.

A. Single Carrier Scenario

For single-carrier scenario, we consider a different maximum average transmit power $p_t = 0.4$W. At first, we study the effect of power allocation and forward delay n on the SINR $\gamma_{\text{AF},2}$ in FD AF relaying scheme in Fig. 4. From the figure, we observe that the curve is uni-modal, namely the SINR is pseudo-concave with respect to the percentage of allocated power to the

relay node, which confirms the statement in Proposition 2. It is also shown that a shorter forward delay n results in a lower SINR. In FD AF relaying, the overall energy for the packet (data size D) transmission is limited by Mp_t and allocated to two blocks (corresponding to the two hops) each with blocklength $M - n$, i.e., $(M - n)(p_S + p_R) = Mp_t$. As a result, a smaller n leads to a lower average power level at source and relay, and consequently a lower received SINR.

Next, we depict in Fig. 5 the different resulting overall error probability with different power allocation ratios for the relay. The reliability performances of different transmission strategies, i.e., direct transmission, FD DF relaying and FD AF relaying, are displayed for comparison. As illustrated in the figure, with an optimized power allocation, both the DF and AF relaying schemes show higher reliability than the direct transmission, which proves the benefits of introducing a relay node. Furthermore, the DF relaying scheme is observed to gain a better performance than AF relaying with different delay n. It should be pointed out that although it has been shown in Fig. 4 that a smaller n results in a lower SINR in AF relaying, a higher reliability is still discovered in Fig. 5 when n is smaller. This is due to the fact that the blocklength $M - n$ for both hops is enlarged when n decreases, which leads to a lower data rate $\frac{D}{M-n}$. This completely compensates the drawbacks of SINR reduction and finally increases the reliability. It implies that in improving the transmission reliability in FD AF relaying, reducing n is preferred than enhancing the SINR at destination. And the convexity of the error probability in FD DF relaying with respect to the power allocation ratio at the relay is also confirmed in Fig. 5, which is proved in the Proposition 1. In addition, in order to depict the performance gain of applying full-duplex techniques, we also show the error probability of half-duplex DF relaying in Fig. 5. Since in HD relaying, half of the transmission period for both source and relay is idle, so that more resource can be saved. To keep a fairness in comparison, we assume the data rate and the available energy for each transmission block are doubled in HD FD relaying. Clearly, we can observe from the figure that introducing full-duplex relaying has an obvious performance gain on reliability. The optimal power allocation ratio differs from that in HD FD relaying, due to the existence of residual loop interference, which indicates the necessity of resource allocation in full-duplex relaying network.

Finally, we discuss the effect of residual interference gain in FD on the system performance. Under varying residual interference gain, the corresponding error probabilities of different transmission strategies are presented in Fig. 6. It is clearly shown in the figure that the error probability
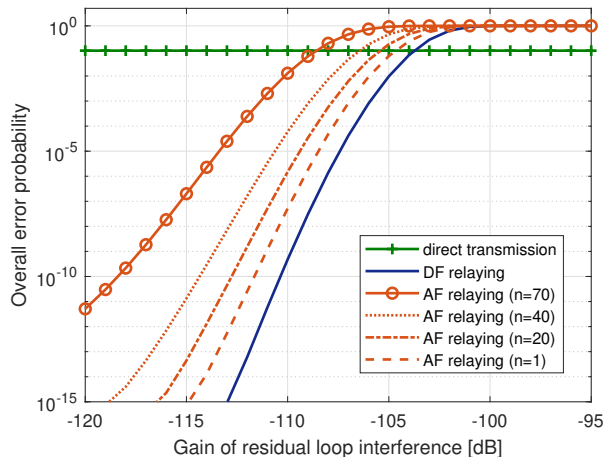
Fig. 6. The impact of the residual loop interference on the achievable reliability in single-carrier scenario.

of direct transmission is not influenced by the residual interference gain and remains constant. Moreover, for both DF relaying and AF relaying, a lower residual interference gain, namely better interference cancellation, always leads to a lower overall error probability. When the residual interference gain is relatively higher, DF and AF relaying schemes have shown a worse performance than the direct transmission, which implies deploying an FD relay node in this case is not worthwhile. Furthermore, in comparison, we find that with varying residual interference gain, the DF relaying always outperforms AF relaying with different delay n. We also notice that when the delay n in AF relaying is smaller, the corresponding error probability drops faster as the residual interference gain decreases. As a result, in the FD AF relaying, a smaller n leads not only to a lower error probability, but also to a higher performance sensitivity with respect to interference cancellation.

B. Multi-Carrier Scenario

So far, we have performed simulations on a single-carrier scenario and validated the proposed algorithms for power allocation. As for the multi-carrier scenario, we have proposed algorithms for joint power and data allocation in both FD DF and FD AF relaying schemes. Before the numerical evaluations, we first introduce three state-of-the-art schemes (as benchmarks), i.e., **direct transmission**, **DF relaying with independently optimized (IO) carriers** and **AF relaying with independently optimized (IO) carriers**, for an explicit comparison:

- For **direct transmission**, we assume all the carriers are deployed with direct link and the data packet to transmit is uniformly allocated to these carriers, i.e., each carrier l is allocated with data amount of $D_l = \frac{D}{N}$ bits. The packet is successfully transmitted only when all data sub-packets via different carriers are correctly decoded.
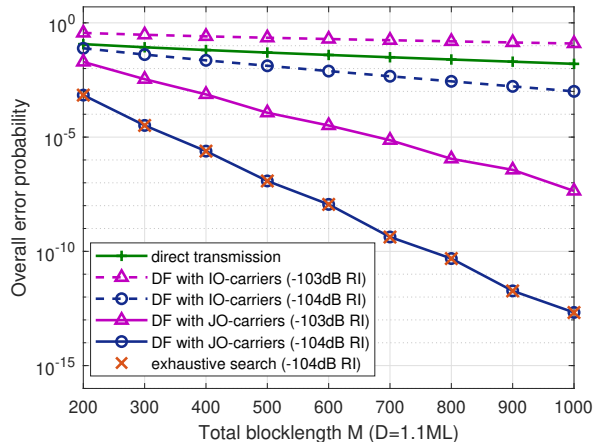
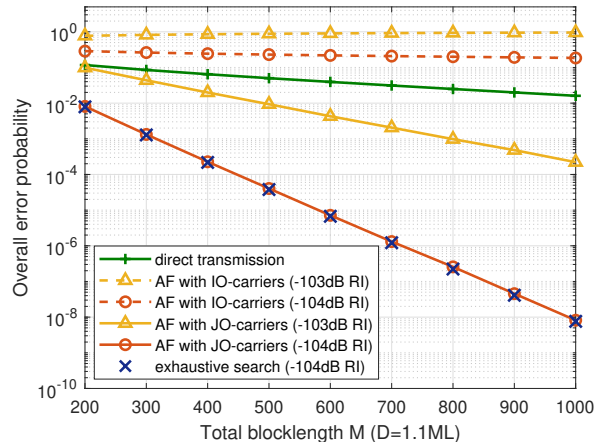Fig. 7. Overall error probability comparison with FD DF relaying scheme and multiple carriers.

Fig. 8. Overall error probability comparison with FD AF relaying scheme and multiple carriers.

- In **DF relaying with IO-carriers**, an FD DF relaying scheme is applied. The resource optimization is independently operated within each carrier, instead of among all carriers. The data packet and the total energy are uniformly divided to N partitions with each assigned to one carrier, so that each carrier is responsible for transmitting a data sub-packet of $\frac{D}{N}$ bits with total energy of $\frac{Mp_t}{N}$. Then, within each carrier, the ratio of power allocated to relay node is optimized based on our proposed power allocation algorithm for single-carrier scenarios.

- In **AF relaying with IO-carriers**, the FD AF relaying is considered. Similarly, the data packet and total energy is equally allocated to all carriers, and each carrier independently optimizes the power assigned to the source and the relay node.

- By contrast, the solutions of joint resource allocation among multi-carriers based on our proposed algorithms are respectively represented by **DF relaying with jointly optimized (JO) carriers** and **AF relaying with jointly optimized (JO) carriers**.

We start with Fig. 7 to evaluate the system enhancement of deploying joint resource allocation in the FD DF relaying scheme and to display the transmission reliability as a function of the latency constraint, namely the total blocklength M. As illustrated in Fig. 6, the error probability for the single-carrier scenario increases as the residual interference gain increases. Therefore, in Fig. 7, we focus on two relatively larger residual interference gains, i.e., -103dB and -104dB, in order to highlight the profits of performing joint resource allocation. Clearly, we can observe from the figure that with a fixed data rate $(D = 1.1ML)$, for all transmission strategies, the overall error probability will be reduced when the total blocklength M becomes larger. This indicates

that a loose latency constraint leads to a higher reliability performance. Moreover, it can also be observed that with IO-carriers, the transmission reliability of DF relaying is relatively lower and even worse than direct transmission (when residual interference gain is -103dB). However, for both residual interference gains, the joint data and power allocation among carriers in our proposed solution has significantly reduced the overall error probability of the network, and the corresponding performance is much better than the direct transmission and DF relaying with IO-carriers. This shows the significant advantage and importance of joint resource allocation over the independent power allocation within each carrier. In addition, from the figure, we also discover that the reliability enhancement from joint resource allocation becomes more significant when the residual interference gain is smaller, which means a joint resource allocation is highly suggested when the interference cancellation techniques get improved. And for both IO-carriers and JO-carriers, the error probability difference between two residual interference gains increases with the total blocklength M. Therefore, it will be more beneficial to improve the interference cancellation techniques when the latency constraint M gets larger.

Next, we study in Fig. 8 the system performance in FD AF relaying with multiple carriers. Similar to the conclusion for FD DF relaying, with a fixed data rate ($D = 1.1ML$), a larger total blocklength M reduces the error probability from all transmission strategies and results in a higher sensitivity of the jointly optimized solution to the residual interference gain. Moreover, the AF relaying solutions based on independent carrier power allocation have shown worse reliability, compared to the direct transmission, while the joint resource allocation has also shown a significant benefit in improving the error probability in AF relaying scheme. It should be mentioned that for both FD DF relaying and FD AF relaying evaluated in Fig. 7 and Fig. 8, we display an exhaustive search result with residual loop interference gain of -104dB for comparison. The exhaustive search is operated via dividing the feasible set by a grid with a sufficiently small step size. The optimal solution is then exhaustively searched by evaluating all the feasible points, which results in a significantly high complexity. In both Fig. 7 and Fig. 8, the corresponding resulting suboptimal solution from our proposed algorithm for multi-carrier scenario is very close to the exhaustively searched result, while only slight differences can be observed, which also illustrates the performance of our iterative algorithms in addressing the joint optimization problems.

Then, in Fig. 9, we compare the DF relaying scheme with AF relaying and direct transmission
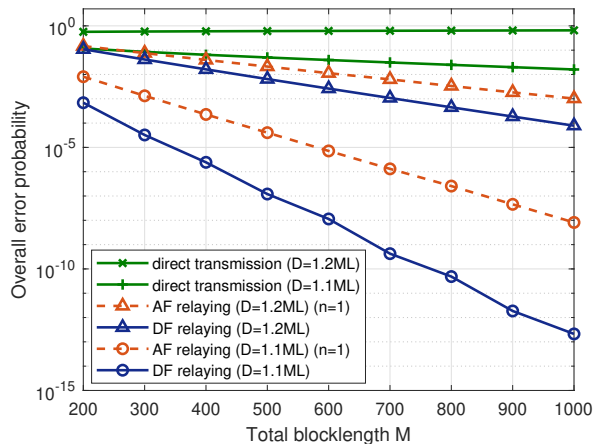
Fig. 9. Overall error probability comparison of multi-carrier DF and AF relaying with respect to total blocklength.
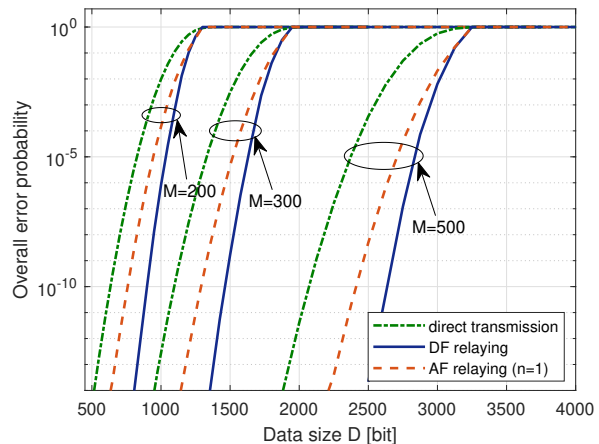

Fig. 10. Overall error probability comparison of multi-carrier DF and AF relaying with respect to total data packet size.

under different data rates. From the figure, we notice that a larger data rate ($D = 1.2ML$) results in a higher error probability for all transmission strategies. Since a larger data rate approaches closer to the Shannon capacity, the corresponding transmission will be more unreliable in the FBL regime. Furthermore, in the multi-carrier scenario, for two different data rates, DF relaying always outperforms AF relaying. And the performance gain of DF relaying compared to AF relaying becomes significant as the total blocklength increases. Thus, for a larger latency constraint M in a multi-carrier scenario, DF relaying should be considered to have a priority to be implemented.

To finalize our numerical evaluations, we show in Fig. 10 the reliability of different transmission strategies as a function of the total data size D. From the figure, it can be recognized that in most cases, the DF relaying shows the best reliability performance, while the AF relaying excels DF relaying only when the data packet size D is relatively large. And a larger data size D leads to a larger data rate and subsequently a higher overall error probability. Moreover, as indicated in the figure, the performance gains of deploying AF relaying (over direct transmission) and of DF relaying over AF relaying significantly increase, when the blocklength M becomes larger (as also shown in Fig. 9) or the data size D gets smaller.

VI. CONCLUSION

In this work, considering FBL regime, we modelled the reliability for a two-hop network applying FD DF and FD AF relaying schemes. Taking a latency constraint and an energy consumption limit into account, we focused on minimizing the overall error probability for both two relaying schemes by optimally designing the power allocation at the source and the relay node. More specifically, with approximated error probability, the power allocation problem

with FD DF relaying is proved to be convex, which is validated in the simulation process. For the error probability minimization in FD AF relaying, we equivalently maximize the ultimately received SINR, which is proved to be a pseudo-concave and makes the equivalent optimization problem pseudo-convex.

Besides, we also extend our work into a multi-carrier scenario, in which a joint data and power allocation is required for the overall error probability minimization. Since the previous power allocation algorithm for single carrier does not work for a joint optimization among carriers, we proposed innovative algorithms respectively for FD DF and FD AF relaying schemes with multiple carriers. In particular, we alternatively treated the SNR, instead of the allocated power, as the variable to be optimized. First, a joint convexity of error probability over SNR and data size is proved. Then, the problem for reliability maximization in FD DF relaying with multiple carriers is reformulated and approximated to a convex problem, based on which an iterative algorithm is proposed to obtain a suboptimal solution. As for FD AF relaying with multiple carriers, we introduce a series of auxiliary variables to facilitate the problem reformulation. Subsequently, a convex approximation and an iterative algorithm is accordingly proposed.

Finally, through simulations, all the algorithms for both FD DF and FD AF relaying schemes and for both single carrier and multiple carriers are validated and compared with direct transmissions. The advantages of introducing both DF and AF relaying node are confirmed. A general better reliability from FD DF relaying is also observed, compared to FD AF relaying. Furthermore, the joint resource allocation among multiple carriers has shown to significantly outperform the independent power allocation within each carrier, and is capable of compensating the system performance when single-carrier optimization cannot satisfy the reliability requirements. In addition, it also indicated that improving interference cancellation techniques is especially beneficial when the latency constraint is loose (the blocklength is larger) or when the joint resource allocation is applied.

Note that although in this paper, we have focused on a simple three-node relaying network, the strategies for the reliability modeling and the minimization of error probability can be extended to more complicated scenarios, e.g., multi-hop relaying network and scenario with imperfect channel information state (CSI). Specifically, our design for the multi-carrier scenario can be extended into a multi-user network combining with the consideration of carrier allocation among users, which is also considered as our future work.

APPENDIX A

PROOF OF INCREASING PROPERTY IN $g_1(\gamma_1)$

The function $g_1(\gamma_1)$ is described as

$$g_1(\gamma_1) = \frac{\gamma_1\left(\gamma_1(\gamma_1+2)-\ln(1+\gamma_1)-\frac{1}{e}\right)}{(\ln 2)^3 V(\gamma_1)^{\frac{3}{2}}(1+\gamma_1)^3} = \frac{\gamma_1(\gamma_1+2)-\ln(1+\gamma_1)-\frac{1}{e}}{(\gamma_1+2)\sqrt{\gamma_1(\gamma_1+2)}}. \tag{61}$$

The first-order derivative of $g_1(\gamma_1)$ is given by

$$\begin{aligned}
g_1'(\gamma_1) &= \frac{2\gamma_1+2-\frac{1}{\gamma_1+1}-(\gamma_1^2+2\gamma_1-\ln(1+\gamma_1)-\frac{1}{e})\frac{2\gamma_1+1}{\gamma_1^2+2\gamma_1}}{(\gamma_1+2)\sqrt{\gamma_1(\gamma_1+2)}} \\
&> \frac{2\gamma_1+2-\frac{1}{\gamma_1+1}-(\gamma_1^2+2\gamma_1)\frac{2\gamma_1+1}{\gamma_1^2+2\gamma_1}}{(\gamma_1+2)\sqrt{\gamma_1(\gamma_1+2)}} \\
&= \frac{\frac{\gamma_1}{\gamma_1+1}}{(\gamma_1+2)\sqrt{\gamma_1(\gamma_1+2)}}.
\end{aligned} \tag{62}$$

Clearly, we have $g_1'(\gamma_1) > 0$ when $\gamma_1 \geq 1$. Namely, the function $g_1(\gamma_1)$ is monotonically increasing with respect to γ_1, $\forall \gamma_1 \geq 1$.

APPENDIX B

PROOF OF INCREASING PROPERTY IN $g_2(\gamma_{i,l})$

The function $g_2(\gamma_{i,l})$ is denoted by

$$g_2(\gamma_{i,l}) = \left((1+\gamma_{i,l})^2-1\right)^2 - \frac{\sqrt{2/M_{\min}}}{Q^{-1}(\varepsilon_{\max})}\sqrt{1-\frac{1}{(1+\gamma_{i,l})^2}} + (2-3(1+\gamma_{i,l})^2)\left(\ln(1+\gamma_{i,l})+\frac{\ln(M_{\min}/2)}{M_{\min}/2}\right). \tag{63}$$

Accordingly, the first-order derivative of function $g_2(\gamma_{i,l})$ can be derived as

$$\begin{aligned}
g_2'(\gamma_{i,l}) &= 4((1+\gamma_{i,l})^2-1)(1+\gamma_{i,l}) + \frac{\frac{\sqrt{2/M_{\min}}}{Q^{-1}(\varepsilon_{\max})}}{(1+\gamma_{i,l})^2\sqrt{\gamma_{i,l}(\gamma_{i,l}+2)}} \\
&\quad -6(1+\gamma_{i,l})\left(\ln(1+\gamma_{i,l})+\frac{\ln(M_{\min}/2)}{M_{\min}/2}\right)+\frac{2-3(1+\gamma_{i,l})^2}{1+\gamma_{i,l}}. \tag{64}
\end{aligned}$$

By ignoring some positive terms, we obtain the inequality

$$\begin{aligned}
g_2'(\gamma_{i,l}) &> 4((1+\gamma_{i,l})^2-1)(1+\gamma_{i,l}) - 6(1+\gamma_{i,l})\left(\ln(1+\gamma_{i,l})+\frac{\ln(M_{\min}/2)}{M_{\min}/2}\right)-3(1+\gamma_{i,l}) \\
&\geq (1+\gamma_{i,l})\underbrace{\left(4(1+\gamma_{i,l})^2-6\ln(1+\gamma_{i,l})-\frac{6}{e}-7\right)}_{g_3(\gamma_{i,l})}. \tag{65}
\end{aligned}$$

The function $g_3(\gamma_{i,l})$ is clearly an increasing function in $\gamma_{i,l}$ when $\gamma_{i,l} \geq 1$, i.e., $g_3(\gamma_{i,l}) \geq g_3(1) = 2.63 > 0$ when $\gamma_{i,l} \geq 1$. Therefore, we have $g_2'(\gamma_{i,l}) > 0$ when $\gamma_{i,l} \geq 1$, and $g_2(\gamma_{i,l})$ is

monotonically increasing in $\gamma_{i,l} \geq 1$.

REFERENCES

[1] Y. Hu, E. Jorswieck, A. Schmeink, "Full-Duplex Relay in High-Reliability Low-latency Networks Operating with Finite Blocklength Codes", *IEEE ISWCS*, Aug. 2019, Oulu, Finland.

[2] C. She, C. Yang, and T. Q. S. Quek, "Radio Resource Management for Ultra-reliable and Low-latency Communications,"*IEEE Commun. Mag.*, vol. 55, no. 6, pp 72-78, Jun. 2017.

[3] C. She, C. Yang, and T. Q. S. Quek,"Cross-layer Optimization for Ultra-reliable and Low-latency Radio Access Networks,"*IEEE Trans. Wireless Commun.*, vol. 17, no. 1, pp. 127-141, Jan. 2018.

[4] O. L. A. Lopez, H. Alves, R. D. Souza and M. Latva-Aho, "Finite Blocklength Error Probability Distribution for Designing Ultra Reliable Low Latency Systems," *IEEE Access*, vol. 8, pp. 107353-107363, 2020.

[5] Mahmood, Nurul Huda, et al. "White paper on critical and massive machine type communication towards 6G." *arXiv preprint arXiv:2004.14146*, 2020.

[6] A. Dogra, R. K. Jha and S. Jain, "A Survey on Beyond 5G Network With the Advent of 6G: Architecture and Emerging Technologies," *IEEE Access*, vol. 9, pp. 67512-67547, 2021.

[7] A. Aijaz and M. Sooriyabandara, "The Tactile Internet for Industries: A Review," *Proc. IEEE*, vol. 107, no. 2, pp. 414–435, Feb. 2019.

[8] 3GPP Release 16. Avaliable: http://www.3gpp.org/release-16.

[9] H. Ren, C. Pan, Y. Deng, M. Elkashlan and A. Nallanathan, "Joint Power and Blocklength Optimization for URLLC in A Factory Automation Scenario," *IEEE Trans. Wireless Commun.*, vol. 19, no. 3, pp. 1786-1801, March 2020.

[10] E. Dosti, U. L. Wijewardhana, H. Alves and M. Latva-aho, "Ultra Reliable Communication via Optimum Power Allocation for Type-I ARQ in Finite Block-length," *IEEE International Conference on Communications (ICC)*, Paris, 2017, pp. 1-6.

[11] Y. Xu, C. Shen, T. Chang, S. Lin, Y. Zhao and G. Zhu, "Energy-Efficient Non-Orthogonal Transmission under Reliability and Finite Blocklength Constraints," *2017 IEEE Globecom Workshops (GC Wkshps)*, Singapore, 2017, pp. 1-6.

[12] J.N. Laneman, D.N.C. Tse and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. Info. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[13] T. Cover and A. Gamal, "Capacity Theorems for The Relay Channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572-584, Sep. 1979.

[14] T. Tran, N. Tran, H. Bahrami, and S. Sastry, "On Achievable Rate and Ergodic Capacity of NAF Multi-Relay Networks with CSI," *IEEE Trans. Commun.*, vol. 62, no. 5, pp. 1490-1502, May 2014.

[15] M. Dai, P. Wang, S. Zhang, B. Chen, H. Wang, and X. Lin, "Survey on Cooperative Strategies for Wireless Relay Channels." *Trans. Emerg. Telecom. Techno.*, vol. 25, no. 9, pp. 926-942, 2014.

[16] Y.S. Choi and H. Shirani-Mehr, "Simultaneous Transmission and Reception: Algorithm, Design and System Level Performance," *IEEE Tran. Wireless Commun.*, vol. 12, pp. 5992–6010, Dec. 2013

[17] G. Liu, F. R. Yu, H. Ji, V. C. M. Leung and X. Li, "In-Band Full-Duplex Relaying: A Survey, Research Issues and Challenges," *IEEE Commun. Surv. Tut.*, vol. 17, no. 2, pp. 500-524, Secondquarter 2015.

[18] L. Zhang, J. Jiang, A. J. Goldsmith and S. Cui, "Study of Gaussian Relay Channels with Correlated Noises," *IEEE Trans. Commun.*, vol. 59, no. 3, pp. 863-876, March 2011.

[19] T. Riihonen, S. Werner and R. Wichman, "Comparison of Full-Duplex and Half-Duplex Modes with a Fixed Amplify-and-Forward Relay," *2009 IEEE Wireless Communications and Networking Conference*, Budapest, Hungary, 2009, pp. 1-5.

[20] T. Riihonen, S. Werner, R. Wichman and E. B. Zacarias, "On the Feasibility of Full-Duplex Relaying in the Presence of Loop Interference," *IEEE Workshop on Signal Processing Advances in Wireless Communications*, Perugia, Italy, 2009, pp. 275-279.

[21] S. Dang, J. P. Coon and G. Chen, "Resource Allocation for Full-Duplex Relay-Assisted Device-to-Device Multicarrier Systems," *IEEE Wireless Commun. Lett.*, vol. 6, no. 2, pp. 166-169, April 2017.

[22] Y. Sun, D. W. K. Ng, Z. Ding and R. Schober, "Optimal Joint Power and Subcarrier Allocation for Full-Duplex Multicarrier Non-Orthogonal Multiple Access Systems," *IEEE Trans. Commun.*, vol. 65, no. 3, pp. 1077-1091, March 2017.

[23] Y. Polyanskiy, H. Poor, and S. Verdu, "Channel Coding Rate in the Finite Blocklength Regime," *IEEE Trans. Inf. Theory*, vol. 56, no. 5, pp. 2307-2359, May 2010.

[24] Y. Hu, J. Gross and A. Schmeink, "On the Capacity of Relaying with Finite Blocklength," *IEEE Trans. Veh. Technol.*, vol. 65, no. 3, pp. 1790-1794, Mar. 2016.

[25] Y. Hu, A. Schmeink and J. Gross, "Optimal Scheduling of Reliability-Constrained Relaying System under Outdated CSI in the Finite Blocklength Regime," *IEEE Trans. Veh. Technol.*, vol. 67, no. 7, pp. 6146-6155, July 2018.

[26] Y. Gu, H. Chen, Y. Li and B. Vucetic, "Ultra-Reliable Short-Packet Communications: Half-Duplex or Full-Duplex Relaying?" *IEEE Wireless Commun. Lett.*, vol. 7, no. 3, pp. 348-351, June 2018.

[27] K. Singh, S. Biswas, M. Ku and M. F. Flanagan, "Transceiver Design for Full-Duplex Ultra-Reliable Low-Latency Communications with Finite Blocklength," *IEEE WCNC*, Seoul, South Korea, 2020.

[28] M. Iwamura, H. Takahashi, and S. Nagata, "Relay Technology in LTE-Advanced," *NTT DOCOMO Techn. J.*, vol. 18, no. 2, pp. 31–36, Jul. 2010

[29] V. Y. F. Tan and M. Tomamichel, "The Third-Order Term in the Normal Approximation for the AWGN Channel," *IEEE Trans. Inf. Theory*, vol. 61, no. 5, pp. 2430-2438, May 2015.

[30] C. She, C. Yang, and T. Q. S. Quek,"Cross-layer Optimization for Ultra-reliable and Low-latency Radio Access Networks,"*IEEE Trans. Wireless Commun.*, vol. 17, no. 1, pp. 127-141, Jan. 2018.

[31] C. Sun, C. She, C. Yang, T. Q. S. Quek, Y. Li and B. Vucetic, "Optimizing Resource Allocation in the Short Blocklength Regime for Ultra-Reliable and Low-Latency Communications," *IEEE Trans. Wireless Commun.*, vol. 18, no. 1, pp. 402-415, Jan. 2019.

[32] S. Boyd and L. Vandenberghe, "Convex Optimization," *Cambridge, U.K.*: Cambridge Univ. Press, 2004.

[33] A. Zappone and E. Jorswieck, "Energy Efficiency in Wireless Networks via Fractional Programming Theory," *Foundations and Trends in Commun. and Inf. Theory*, vol. 11, no. 3-4, pp. 185-196, June 2015.

[34] B. Suard, G. Xu, H. Liu and T. Kailath, "Uplink Channel Capacity of Space-Division-Multiple-Access Schemes," *IEEE Trans. Inf. Theory*, vol. 44, no. 4, pp. 1468-1476, July 1998.

[35] C. Wang, Y. Fan, J. S. Thompson and H. V. Poor, "A Comprehensive Study of Repetition-Coded Protocols in Multi-User Multi-Relay Networks," *IEEE Trans. Wireless Commun.*, vol. 8, no. 8, pp. 4329-4339, August 2009.

[36] M. Nassar and B. L. Evans, "Low Complexity EM-Based Decoding for OFDM Systems with Impulsive Noise," *Conference Record of the Forty Fifth Asilomar Conference on Signals, Systems and Computers (ASILOMAR)*, Pacific Grove, CA, USA, 2011, pp. 1943-1947.

[37] X. Yuan, T. Yang, Y. Hu, J. Xu and A. Schmeink, "Trajectory Design for UAV-Enabled Multiuser Wireless Power Transfer With Nonlinear Energy Harvesting," *IEEE Trans. Wireless Commun.*, vol. 20, no. 2, pp. 1105-1121, Feb. 2021.

[38] S. Boyd. *EE364b Convex Optimization II, Course Notes*, accessed on Jun. 29, 2017. [Online]. Available: http://www.stanford.edu/class/ ee364b/

[39] T. S. Rappaport, "Wireless Communications: Principles and Practice," 2nd ed. Prentice Hall, 2002.