Joint Design of UAV Trajectory and Directional Antenna Orientation in UAV-Enabled WPT Networks

Xiaopeng Yuan[†,‡], Yulin Hu[†,‡] and Anke Schmeink[‡]
[†]School of Electronic Information, Wuhan University, China, Email: $yulin.hu$@whu.edu.cn
[‡]ISEK Research Area, RWTH Aachen, Germany, Email: $yuan|schmeink$@isek.rwth-aachen.de

***Abstract*—We investigate an unmanned aerial vehicle (UAV)-enabled wireless power transfer (WPT) network. A UAV with a directional antenna array is operated to simultaneously wirelessly transfer energy to multiple ground sensor nodes (SNs). We consider a nonlinear energy harvesting (EH) model and a directional antenna structure of uniform linear array (ULA) where an analog directional beamforming scheme is applied. Taking the fairness issue into account, we formulate a problem aiming at maximizing the minimum harvested energy among all SNs during a fixed time period, via jointly optimizing the UAV trajectory and the orientation of the directional antenna on UAV. Then, we construct a tight convex approximation for the optimization problem, based on which the problem is solved via a proposed iterative algorithm and the objective converges to an efficient suboptimal solution. Finally, we provide numerical results to validate our proposed algorithm and to highlight the benefits of deploying directional antenna in WPT networks.**

***Index Terms*—UAV, nonlinear energy harvesting (EH), directional antenna, trajectory design, joint optimization.**

I. Introduction

Recently, unmanned aerial vehicles (UAVs) have enabled numerous applications in various scenarios and has consequently stimulated abundant investigations [1], [2]. In UAV-aided wireless networks, due to the high likelihood of existence of line-of-sight (LoS) dominant air-ground channels, considerable benefits have been discovered from system evaluations, in comparison to the terrestrial cellular networks [3]. In particular, the system performance can be enhanced by designing the deployment position and the trajectory of UAV [4]. Implementing UAV as a mobile base station, researchers have made contributions in improving the coverage [5] and the channel capacity [6]. Furthermore, since the UAV is actually experiencing a changing communication environment when we deploy its mobility, a joint design of the UAV trajectory and resource allocation regarding the variant channel states is generally recommended in UAV-assisted networks. For instance, a joint power control has been performed with a UAV trajectory design for secure communications [7] and for a higher reliability in relaying networks [8].

Besides, incorporating UAVs into wireless power transfer (WPT) networks also attracts much attention in research [9]. The WPT technologies have been confirmed to be capable of prolonging the lifetime of network devices and sustaining the network [10], [11]. By combining WPT technologies with UAV and applying UAV as an energy supporter, an improved WPT performance can be expected. In literature, much work [12]–[14] has been done in studying UAV trajectory design in pure WPT networks. To accurately evaluate the WPT performance, two models characterizing the nonlinearity in energy harvesting (EH) have been proposed, i.e., a statistic model [15], which is obtained by curve fitting on statistic data, and an analytical model [16], which is formulated from the structure of conversion circuit with a diode. However, it is also observed from [14] that with practical EH setups, the harvested power is largely constricted by the significant attenuation in signal transmission.

On the other hand, directional antenna can evidently stimulate a significantly amplified antenna gain [17], which definitely helps in the WPT process. Without channel state information, the authors in [18] have adopted an analog directional antenna and studied a UAV-enabled WPT network with a two-user scenario. However, the applied model is a flat-top one, which approximates the gain in 3dB beamwidth to a constant and is clearly lack of accuracy, and the obtained conclusion only fits the two-user scenario. To the best of our knowledge, for a multi-user WPT network assisted by a UAV with directional antenna, it is still missing in the literature how to efficiently design the UAV trajectory and antenna orientation, especially when considering an accurate pattern model of the directional antenna.

In this work, we consider a UAV-enabled multi-user WPT network, where the UAV operates with a directional antenna utilizing a uniform linear array (ULA) and performing analog beamforming. In addition, an analytical nonlinear EH model [16] is taken into account for characterizing the WPT performance. For *the first time*, we consider a joint design, i.e., jointly (instead of alternately) optimizing the UAV trajectory and directional antenna orientation, aiming at maximizing the minimum harvested energy among all users. By constructing a convex approximation of the focused problem, we address the joint optimization problem with an iterative algorithm which guarantees a convergence behaviour of the WPT performance and eventually results in an efficient suboptimal solution.

The rest part of this paper is organized as follows. We state the system model and the problem in Section II. Section III provides a convex approximation to the problem and propose an iterative solution, which is further numerically evaluated in Section IV. Finally, we conclude our work in Section V.

II. Preliminary

A. System Model

We study a UAV-enabled wireless sensor network, where multiple sensor nodes (SNs) are distributed on the ground surface, as shown in Fig. 1. We assume that there are K SNs and denote the corresponding horizontal position of SN k by

Y. Hu is the corresponding author.

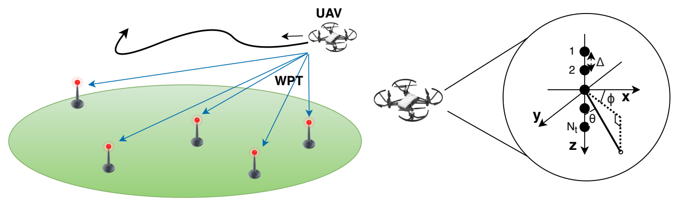

Fig. 1. An example of considered UAV-enabled WPT network.

$(w_{x,k}, w_{y,k}), k \in \mathcal{K} \triangleq \{1, ..., K\}$. To prolong the lifetimes of these SNs and enrich their battery energy, a single UAV at a fixed altitude H is deployed as an energy supply and responsible for wirelessly transferring energy to the SNs via radio frequency (RF), as displayed in Fig. 1. We consider a fixed time period, $\mathcal{T} \triangleq [0, T]$ with $T > 0$, for the wireless charging process. Note that the length T of the charging period is pre-designed, perhaps according to the available energy at the UAV or an external time scheduling policy. Then, we uniformly discretize the charging time duration. Namely the whole time period is divided into N time slots each with length $\delta T = \frac{T}{N}$. Note that the time resolution δT is sufficiently small so that in each time slot the position of UAV can be assumed to be static. We denote by $(x[n], y[n])$ the horizontal position of UAV at height H in time slot $n \in \mathcal{N} \triangleq \{1, ..., N\}$. The moving speed of UAV is constrained by a maximum value V, and the speed constraint is accordingly formulated as $\forall n \in \mathcal{N},\ n \neq N$,

$$(x[n+1] - x[n])^2 + (y[n+1] - y[n])^2 \leq V^2 \delta T^2, \tag{1}$$

which representing that the distance between UAV positions in each two neighbour time slots should be constricted by the maximum movement $V\delta T$ within a time slot.

Moreover, we assume the UAV operates with a directional antenna, with which the channel gain can be significantly enhanced in a certain direction. We adopt a directional antenna with a structure of uniform linear array (ULA), where N_t isotropic antenna elements are uniformly placed along a vertical straight line with equal spacing Δ. An example of ULA directional antenna is shown in Fig. 1. As commonly selected in ULA [19], the antenna spacing Δ is designed to be half of the wavelength, i.e., $\Delta = \frac{\lambda}{2}$ where λ is the wavelength of the centre carrier frequency for WPT. We focus on an analog directional beamforming, which has already attracted much attentions in diverse applications due to its low cost and low power consumption. Note that with a sufficiently small δT, the orientation scheme of directional antenna can also be treated as constant. Thus, we denote by $\theta_0[n] \in [0, \frac{\pi}{2})$ in time slot n the elevation angle of the direction for power concentration in the directional beamforming design of the ULA, namely a maximum power gain (main lobe) will be observed at the direction with elevation angle $\theta_0[n]$ in time slot n. Then, according to [20], with the optimal analog beamforming, the resulted antenna gain to SN k is given by

$$g_{\mathrm{A}}(\theta_k[n], \phi, \theta_0[n]) = \frac{1}{N_t}\left(\frac{\sin(N_t \pi \varphi_k[n])}{\sin(\pi \varphi_k[n])}\right)^2, \tag{2}$$

where $\theta_k[n]$ and ϕ represent the elevation angle and azimuth angle from UAV to SN k in time slot n, respectively, while $\varphi_k[n] = \frac{1}{2}(\cos\theta_k[n] - \cos\theta_0[n])$ is the auxiliary variable. Note

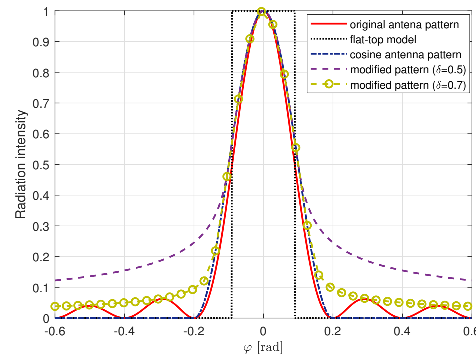

Fig. 2. Comparison of different antenna pattern approximation ($N_t = 5$).

that with ULA structure along a straight line, the antenna gain is actually independent of the azimuth angle ϕ. Therefore, we can simply drop the variable ϕ and rewrite the antenna gain as $g_{\mathrm{A}}(\theta_k[n], \theta_0[n])$. Then, to obtain the antenna gain, only the formulation of $\theta_k[n]$ is required, which can be obtained as

$$\theta_k[n] = \arccos\left(\frac{H}{d_k[n]}\right), \tag{3}$$

where $d_k[n]$ is the distance from UAV to SN k in time slot n:

$$d_k[n] = \sqrt{(x[n] - w_{x,k})^2 + (y[n] - w_{y,k})^2 + H^2}. \tag{4}$$

Accordingly, we have the auxiliary variable $\varphi_k[n] = \frac{1}{2}(\frac{H}{d_k[n]} - \cos\theta_0[n])$. Since the variable $\theta_0[n]$ for directional antenna orientation only appears in $\varphi_k[n]$ within a term of $\cos\theta_0[n]$, we substitute the variables $\theta_0[n]$ with a new variable $\mu[n] = \cos\theta_0[n] \in (0, 1]$ for simplification and reformulate $\varphi_k[n]$ as

$$\varphi_k[n] = \frac{1}{2}(\frac{H}{d_k[n]} - \mu[n]). \tag{5}$$

Furthermore, the antenna gain given in (2) has sine functions in both the numerator and denominator, and is thus multimodal and not tractable in performance analysis [21]. Therefore, an approximation of the directional antenna gain with both accuracy and tractability is highly recommended. For this purpose, the authors in [21] have proposed a cosine antenna pattern approximation, as shown in Fig. 2, in which the main lobe in antenna pattern is represented by a square of cosine function while the remaining side lobes are ignored. Note that antenna gain is equal to antenna pattern multiplied by N_t. In our problem, we adopt a modified cosine antenna pattern approximation $\mathcal{F}_A$ considering both the main lobe and side lobes in antenna pattern, and approximate the antenna gain as

$$\begin{aligned} &g_{\text{A-mc}}(\theta_k[n], \theta_0[n]) = N_t \mathcal{F}_{\mathrm{A}}(\varphi_k[n]) \\ &= \begin{cases} N_t \cos^2\left(\frac{N_t \pi}{2}\varphi_k[n]\right), & \text{when } |\varphi_k[n]| < \frac{\delta}{N_t}, \\ \frac{N_t \cos^2(\frac{\delta\pi}{2})}{\sqrt{2N_t\pi\tan(\frac{\delta\pi}{2})(|\varphi_k[n]| - \frac{\delta}{N_t}) + 1}}, & \text{when } |\varphi_k[n]| \geq \frac{\delta}{N_t}, \end{cases} \end{aligned} \tag{6}$$

where $\delta \in (0, 1)$ is the modification factor. The introduction of δ makes that the modified cosine antenna pattern can have an extremely small value but always be positive. Distinctly, when δ goes to 1, the modified cosine antenna pattern approaches to the proposed cosine pattern in [21], as implied in Fig. 2.

Then, with the formulated antenna gain, we start to model the channel gain between UAV and SNs. In UAV-related wireless communications, the air-to-ground links have a much higher probability to yield LoS and thus, are generally dominated by a

LoS component [3]. Therefore, as broadly performed, we adopt a free-space channel model for all channels between the UAV and SNs. In time slot n, the channel power gain from UAV to an SN k is represented by

$$g_{\text{ch},k}[n] = \frac{\beta_0 g_{\text{A-mc}}(\theta_k[n], \theta_0[n])}{(d_k[n])^2} = \frac{\beta_0 N_t \mathcal{F}_A(\varphi_k[n])}{(d_k[n])^2}, \quad (7)$$

where β_0 is the channel gain at a reference distance of unit meter with an isotropic antenna, $d_k[n]$ is a distance described in (4). Denote by P the constant transmit power of UAV for WPT. Then, the received RF power of SN k at time t is

$$Q_k[n] = g_{\text{ch},k}[n]P = \frac{\beta_0 P N_t \mathcal{F}_A(\varphi_k[n])}{(d_k[n])^2}. \quad (8)$$

During energy harvesting procedure, the received RF signal will be at first converted into a direct-current (DC) signal and then harvested into the battery set at SNs. The conversion is clearly a nonlinear process [22]. Due to the significant attenuation of signal and relatively large distance in wireless transmission, the received RF signal has a generally small power, which motivates us to adopt a small signal model [22] to process the nonlinearity of rectifier (diode) during DC conversion at SNs. According to [16], by performing a Taylor expansion on the nonlinear voltage function over input current in diode, the output current I_{out} for harvesting can be characterized by a nonlinear implicit function $I_{\text{out}}(Q)$ with respect to the received RF power Q. More specifically, as displayed in Eq.(6) in [16], we have

$$e^{\frac{R_{\text{L}} I_{\text{out}}(Q)}{n_0 v_t}} (I_{\text{out}}(Q) + I_{\text{s}}) \approx \sum\nolimits_{j=0}^{n'_o} \alpha_j Q^j, \quad (9)$$

where R_L, n_0, v_t and I_s respectively denote the load resistance, ideality factor, thermal voltage and the reverse bias saturation current at diode, while n'_o is the truncation order. Afterwards, the charged (harvested) power P_{ch} can be directly expressed by the output current $I_{\text{out}}(Q)$, i.e., $P_{\text{ch}} = (I_{\text{out}}(Q))^2 R_L$, which implies that the charged power P_{ch} is also an implicit nonlinear function of received RF power Q, denoted by $\mathcal{F}_{\text{nl}}(Q)$. Therefore, in time slot n, the harvested power $P_{\text{ch},k}[n]$ at SN k is denoted by

$$P_{\text{ch},k}[n] = \mathcal{F}_{\text{nl}}(Q_k[n]) \triangleq (I_{\text{out}}(Q_k[n]))^2 R_L. \quad (10)$$

Although an explicit expression of charged power $P_{\text{ch},k}(t)$ cannot be derived out, we can apply the convexity of this nonlinear process in $\mathcal{F}_{\text{nl}}$, which has been already proved in [16]. In particular, let $u[n] = \frac{\beta_0 P N_t}{Q_k[n]} = \frac{(d_k[n])^2}{\mathcal{F}_A(\varphi_k[n])}$, and $Q_k[n]$ described in (8) can be reformulated as $Q_k[n] = Q_k^*(u) = \frac{\beta_0 P N_t}{u}$. Then, with the first-order and second-order derivatives of $Q_k^*(u)$ given by $\dot{Q}_k^*(u)$ and $\ddot{Q}_k^*(u)$, the following inequality can be proved to hold

$$\ddot{Q}_k^*(u) Q_k^*(u) - \left(\dot{Q}_k^*(u)\right)^2 = \frac{\beta_0^2 P^2 N_t^2}{u^4} > 0. \quad (11)$$

According to [16], satisfying the condition $\ddot{Q}_k^*(u) Q_k^*(u) - (\dot{Q}_k^*(u))^2 \geq 0$ definitely makes the function $\mathcal{F}_{\text{nl}}(Q_k[n]) = \mathcal{F}_{\text{nl}}(Q_k^*(u))$ be convex with respect to u. In other words, for our considered network, $P_{\text{ch},k}[n] = \mathcal{F}_{\text{nl}}(Q_k[n])$ is convex in $\frac{(d_k[n])^2}{\mathcal{F}_A(\varphi_k[n])}$, which facilitates us in analytically modeling and optimizing the network performance.

B. Problem Statement

To fully deploy the WPT UAV and to improve the energy harvesting behaviour, all SNs can simultaneously accept the wirelessly transferred power for energy harvesting during the whole charging period $\mathcal{T}$. As a result, the total harvested energy at SN k is given by

$$E_k(\{x[n], y[n], \mu[n]\}) = \sum_{n=1}^{N} \mathcal{F}_{nl}\left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k[n])}{d_k[n]^2}\right) \delta T. \quad (12)$$

While taking the fairness among SNs into account and focusing on enhancing the WPT performance, we aim at jointly designing the UAV trajectory $\{x[n], y[n]\}$ and the directional antenna orientation $\theta_0[n]$ (alternatively $\mu[n]$) to maximize the minimal harvested energy among all SNs during the charging period $\mathcal{T}$. Thus, the considered optimization problem can be formulated as

$$\text{(OP)}: \max_{\{x[n], y[n], \mu[n]\}} \min_k \{E_k(\{x[n], y[n], \mu[n]\})\} \quad (13a)$$

$$\text{s.t.} \quad 0 < \mu[n] \leq 1, \ \forall n \in \mathcal{N}, \quad (13b)$$

$$(1).$$

Next, with assistance of an auxiliary variable E, the original problem (OP) is equivalently reformulated as

$$\text{(P1)}: \max_{\{x[n], y[n], \mu[n]\}, E} E \quad (14a)$$

$$\text{s.t.} \ E_k(\{x[n], y[n], \mu[n]\}) \geq E, \forall k \in \mathcal{K}, \quad (14b)$$

$$(1), \ (13b).$$

It should be pointed out that both the (OP) and (P1) are non-convex, as the objective in (OP) is non-concave and the first constraint in (P1) is not convex. Thus, the problem cannot be directly optimally solved via convex optimization techniques.

III. Proposed Joint Optimization Algorithm

To address the problem, we in this section construct a tight convex approximation and subsequently propose an iterative algorithm for the problem.

A. Convex Approximation

By observing the problem (P1), we have that to construct a convex approximation of (P1), the constraint (14b) is supposed to be approximated to a convex one. Namely we are required to find a concave function $P_{k,n}^{(r)}$ for the harvested power $\mathcal{F}_{nl}\left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k[n])}{d_k[n]^2}\right)$, so that $\mathcal{F}_{nl}\left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k[n])}{d_k[n]^2}\right) \geq P_{k,n}^{(r)}(x[n], y[n], \mu[n])$ and the equality holds at a local point $(x^{(r)}[n], y^{(r)}[n], \mu^{(r)}[n])$.

Recall that as discussed in Section II, the function $\mathcal{F}_{\text{nl}}\left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k[n])}{d_k[n]^2}\right)$ is convex with respect to $\frac{d_k[n]^2}{\mathcal{F}_{\text{A}}(\varphi_k[n])}$. Based on the property of convex functions, for any pair of k and n, we can obtain an inequality

$$\mathcal{F}_{\text{nl}}\left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k[n])}{d_k[n]^2}\right) \geq -A_{1,k,n}^{(r)} \left(\frac{d_k[n]^2}{\mathcal{F}_{\text{A}}(\varphi_k[n])}\right) + A_{2,k,n}^{(r)}, \quad (15)$$

where the positive constants $A_{1,k,n}^{(r)}$ and $A_{2,k,n}^{(r)}$ are given by

$$A_{1,k,n}^{(r)} = \mathcal{F}'_{nl} \left(\frac{\beta_0 P N_t \mathcal{F}_{\text{A}}(\varphi_k^{(r)}[n])}{d_k^{(r)}[n]^2}\right) \frac{\beta_0 P N_t d_k^{(r)}[n]^4}{\left(\mathcal{F}_{\text{A}}(\varphi_k^{(r)}[n])\right)^2}, \quad (16)$$

$$A^{(r)}_{2,k,n}=\mathcal{F}_{nl}\left(\frac{\beta_0 P N_t \mathcal{F}_{\rm A}(\varphi^{(r)}_k[n])}{d^{(r)}_k[n]^2}\right)+A^{(r)}_{1,k,n}\left(\frac{d^{(r)}_k[n]^2}{\mathcal{F}_{\rm A}(\varphi^{(r)}_k[n])}\right), \quad (17)$$

and the terms $d^{(r)}_k[n]$ and $\varphi^{(r)}_k[n]$ are respectively equal to the obtained values of $d_k[n]$ and $\varphi_k[n]$ at local point $(x^{(r)}[n], y^{(r)}[n], \mu^{(r)}[n])$.

Then, according to (6), the function $\mathcal{F}_{\rm A}(\varphi_k[n])$ is guaranteed to be positive. Based on the inequality of arithmetic and geometric means, i.e., $ab \le \frac{1}{2}a^2 + \frac{1}{2}b^2$ holds $\forall a, b \in \mathcal{R}^+$ and the equality holds when $a = b$, we can obtain from (15)

$$\begin{aligned}&\mathcal{F}_{\rm nl}\left(\frac{\beta_0 P N_t \mathcal{F}_{\rm A}(\varphi_k[n])}{d_k[n]^2}\right)\ge -A^{(r)}_{1,k,n}\left(\frac{d_k[n]^2}{\mathcal{F}_{\rm A}(\varphi_k[n])}\right)+A^{(r)}_{2,k,n}\\ &= -A^{(r)}_{1,k,n}\frac{1}{F^{(r)}_{1,k,n}}\underline{F^{(r)}_{1,k,n}d_k[n]^2}\cdot\underline{\frac{1}{\mathcal{F}_{\rm A}(\varphi_k[n])}}+A^{(r)}_{2,k,n}\\ &\ge -\frac{A^{(r)}_{1,k,n}}{F^{(r)}_{1,k,n}}\underline{\frac{1}{2}(F^{(r)}_{1,k,n}d_k[n]^2)^2}-\frac{A^{(r)}_{1,k,n}}{F^{(r)}_{1,k,n}}\underline{\frac{1}{2(\mathcal{F}_{\rm A}(\varphi_k[n]))^2}}+A^{(r)}_{2,k,n}, \quad (18)\end{aligned}$$

where the positive constant $F^{(r)}_{1,k,n}$ is given by $F^{(r)}_{1,k,n} = \frac{1}{\mathcal{F}_{\rm A}(\varphi^{(r)}_k[n])}\frac{1}{d^{(r)}_k[n]^2}$ and assures that the equality in (18) holds at the local point $(x^{(r)}[n], y^{(r)}[n], \mu^{(r)}[n])$.

Note that by observing the approximation results in (18), we find that the concave approximation $P^{(r)}_{k,n}$ can be completed when a tight upper-bound convex approximation of $\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2}$ can be obtained. Towards this end, we first introduce the Lemma 1, to facilitate the following approximations.

Lemma 1. *For a continuous function $h(\gamma)$ $(\gamma \in \mathcal{D})$, where $\mathcal{D}$ is a continuous domain, and a given value $\gamma_0 \in \mathcal{D}$, if $h''(\gamma) \le c$ $(c \ge 0)$, $\forall \gamma \in \mathcal{D}$, then we have the following inequality holds that $\forall \gamma \in \mathcal{D}$,*

$$h(\gamma) \le \frac{c}{2}\gamma^2+(h'(\gamma_0)-c\gamma_0)\gamma+h(\gamma_0)-\gamma_0 h'(\gamma_0)+\frac{c}{2}\gamma_0^2, \quad (19)$$

and the equality holds when $\gamma = \gamma_0$.

Then, we focus on the function $\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2}$. According to the definition in (6), we have $\left(\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2}\right)''_{\varphi_k[n]} < \frac{5N_t^2\pi^2}{\cos^4(\frac{\delta\pi}{2})}$. According to Lemma 1, we can obtain that

$$\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2} \le B^{(r)}_{1,k,n}(\varphi_k[n])^2 + B^{(r)}_{2,k,n}(\varphi_k[n]) + B^{(r)}_{3,k,n}, \quad (20)$$

where the constants $B^{(r)}_{1,k,n}$, $B^{(r)}_{2,k,n}$ and $B^{(r)}_{3,k,n}$ are given by

$$B^{(r)}_{1,k,n}=\frac{5N_t^2\pi^2}{2\cos^4(\frac{\delta\pi}{2})} > 0, \quad (21)$$

$$B^{(r)}_{2,k,n}=\frac{-2\mathcal{F}'_{\rm A}(\varphi^{(r)}_k[n])}{\left(\mathcal{F}_{\rm A}(\varphi^{(r)}_k[n])\right)^3} - 2B^{(r)}_{1,k,n}\varphi^{(r)}_k[n], \quad (22)$$

$$B^{(r)}_{3,k,n}=\frac{1}{(\mathcal{F}_{\rm A}(\varphi^{(r)}_k[n]))^2}-B^{(r)}_{2,k,n}\varphi^{(r)}_k[n]+B^{(r)}_{1,k,n}(\varphi^{(r)}_k[n])^2. \quad (23)$$

Note that $B^{(r)}_{1,k,n} > 0$ and that $\mu[n] \in (0, 1]$ makes $3-2\mu[n] > 0$ hold. By combining the definition of $\varphi_k[n]$ in (5) with the approximation in (20), based on the inequality of arithmetic and geometric means, we have the inequality (24) (on the top of next page) holds. The positive constant $F^{(r)}_{2,k,n}$ is defined as $F^{(r)}_{2,k,n} = \frac{H}{d^{(r)}_k[n]} \cdot \frac{1}{3-2\mu^{(r)}[n]} > 0$, and guarantees the equality in (24) holds at the local point $(x^{(r)}[n], y^{(r)}[n], \mu^{(r)}[n])$. Clearly, the function $h^{(r)}_{2,k,n}(\mu[n])$ is convex in $\mu[n]$, while $h^{(r)}_{1,k,n}(d_k[n])$ is convex in $d_k[n]$ however not jointly convex in $x[n]$ and $y[n]$ according to (4). Therefore, we next focus on the function $h^{(r)}_{1,k,n}(d_k[n])$ and treat the term $(x[n]-w_{x,k})^2+(y[n]-w_{y,k})^2$ as a variable γ, with which we have $h^{(r)}_{1,k,n}(d_k[n]) = h_0(\gamma) = \frac{B^{(r)}_{1,k,n}}{4}(1 + \frac{1}{2F^{(r)}_{2,k,n}})\frac{H^2}{\gamma+H^2} + \frac{2B^{(r)}_{2,k,n}-3B^{(r)}_{1,k,n}}{4}\frac{H}{\sqrt{\gamma+H^2}}$ $(\gamma \ge 0)$. With a single variable γ, it can be easily proved that the second-order derivative of $h_0(\gamma)$ satisfies $h''_0(\gamma) \le \frac{B^{(r)}_{1,k,n}}{4}(1+\frac{1}{2F^{(r)}_{2,k,n}})\frac{2}{H^4} + \frac{3|2B^{(r)}_{2,k,n}-3B^{(r)}_{1,k,n}|}{16H^4} \triangleq C^{(r)}_{1,k,n}$. Since $C^{(r)}_{1,k,n} > 0$, according to Lemma 1, we can get

$$\begin{aligned}&h^{(r)}_{1,k,n}(d_k[n]) \;=\; h_0((x[n]-w_{x,k})^2+(y[n]-w_{y,k})^2)\\ &\le\; \frac{C^{(r)}_{1,k,n}}{2}((x[n]-w_{x,k})^2+(y[n]-w_{y,k})^2)^2\\ &\quad+C^{(r)}_{2,k,n}((x[n]-w_{x,k})^2+(y[n]-w_{y,k})^2) + C^{(r)}_{3,k,n}\\ &\triangleq\; h^{(r)}_{3,k,n}(x[n], y[n]), \end{aligned} \quad (25)$$

where $C^{(r)}_{2,k,n}$ and $C^{(r)}_{3,k,n}$ are obtained based on Lemma 1 and function $h_0(\gamma)$. For function $h^{(r)}_{3,k,n}(x[n], y[n])$, different sign of the constant $C^{(r)}_{2,k,n}$ will lead to different convexity. Thus, we discuss two cases, i.e., case $C^{(r)}_{2,k,n} \ge 0$ and case $C^{(r)}_{2,k,n} < 0$. When $C^{(r)}_{2,k,n} \ge 0$, the function $h^{(r)}_{3,k,n}(x[n], y[n])$ will be convex with respect to $x[n]$ and $y[n]$. In this case, we define the approximation for $h^{(r)}_{1,k,n}(d_k[n])$ as $h^{(r)}_{4,k,n}(x[n], y[n]) \triangleq h^{(r)}_{3,k,n}(x[n], y[n])$. When $C^{(r)}_{2,k,n} < 0$, the term $C^{(r)}_{2,k,n}((x[n]-w_{x,k})^2 + (y[n]-w_{y,k})^2)$ will be concave in $x[n]$ and $y[n]$. According to the property of concave functions, we have

$$\begin{aligned}&h^{(r)}_{1,k,n}(d_k[n]) \;\le\; h^{(r)}_{3,k,n}(x[n], y[n])\\ &\le\; \frac{C^{(r)}_{1,k,n}}{2}((x[n]-w_{x,k})^2+(y[n]-w_{y,k})^2)^2\\ &\quad+C^{(r)}_{2,k,n}(x^{(r)}[n]-w_{x,k})(2x[n]-x^{(r)}[n]-w_{x,k})\\ &\quad+C^{(r)}_{2,k,n}(y^{(r)}[n]-w_{y,k})(2y[n]-y^{(r)}[n]-w_{y,k})+C^{(r)}_{3,k,n}\\ &\triangleq\; h^{(r)}_{4,k,n}(x[n], y[n]), \end{aligned} \quad (26)$$

where $h^{(r)}_{4,k,n}(x[n], y[n])$ is clearly a convex function.

With the defined $h^{(r)}_{4,k,n}(x[n], y[n])$ from the above discussions and recalling the inequality in (24), we can obtain a convex approximation for function $\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2}$

$$\begin{aligned}&\frac{1}{(\mathcal{F}_{\rm A}(\varphi_k[n]))^2} \le h^{(r)}_{1,k,n}(d_k[n]) + h^{(r)}_{2,k,n}(\mu[n])\\ &\le\; h^{(r)}_{4,k,n}(x[n], y[n]) + h^{(r)}_{2,k,n}(\mu[n]). \end{aligned} \quad (27)$$

By combining (27) with the inequality in (18), we have the final concave approximation for the harvested power as

$$\begin{aligned}&\mathcal{F}_{\rm nl}\left(\frac{\beta_0 P N_t f_{\rm A}(\varphi_k[n])}{d_k[n]^2}\right) \ge -A^{(r)}_{1,k,n}\frac{F^{(r)}_{1,k,n}d_k[n]^4}{2}\\ &-A^{(r)}_{1,k,n}\frac{h^{(r)}_{4,k,n}(x[n], y[n])+h^{(r)}_{2,k,n}(\mu[n])}{2F^{(r)}_{1,k,n}}+A^{(r)}_{2,k,n}\\ &\triangleq P^{(r)}_{k,n}(x[n], y[n], \mu[n]), \end{aligned} \quad (28)$$

$$
\begin{aligned}
\frac{1}{(\mathcal{F}_{\mathrm{A}}(\varphi_k[n]))^2} &\leq \frac{B^{(r)}_{1,k,n}}{4}\frac{H^2}{d_k[n]^2}+\frac{2B^{(r)}_{2,k,n}-3B^{(r)}_{1,k,n}}{4}\frac{H}{d_k[n]}+\frac{B^{(r)}_{1,k,n}}{4F^{(r)}_{2,k,n}}\underline{\underline{F^{(r)}_{2,k,n}(3-2\mu[n])\cdot\frac{H}{d_k[n]}}}+\frac{B^{(r)}_{1,k,n}}{4}\mu[n]^2-\frac{B^{(r)}_{2,k,n}}{2}\mu[n]+B^{(r)}_{3,k,n} \\
&\leq \underbrace{\frac{B^{(r)}_{1,k,n}}{4}\frac{H^2}{d_k[n]^2}+\frac{2B^{(r)}_{2,k,n}-3B^{(r)}_{1,k,n}}{4}\frac{H}{d_k[n]}+\frac{B^{(r)}_{1,k,n}}{4F^{(r)}_{2,k,n}}\underline{\underline{\frac{H^2}{2d_k[n]^2}}}}_{h^{(r)}_{1,k,n}(d_k[n])}+\underbrace{\frac{B^{(r)}_{1,k,n}}{4F^{(r)}_{2,k,n}}\underline{\underline{\frac{\left(F^{(r)}_{2,k,n}(3-2\mu[n])\right)^2}{2}}}+\frac{B^{(r)}_{1,k,n}}{4}\mu[n]^2-\frac{B^{(r)}_{2,k,n}}{2}\mu[n]+B^{(r)}_{3,k,n}}_{h^{(r)}_{2,k,n}(\mu[n])}. \quad (24)
\end{aligned}
$$

where function $P^{(r)}_{k,n}(x[n],y[n],\mu[n])$ is concave and the inequality holds for any feasible point $(x[n],y[n],\mu[n])$. Note that in each step of approximations, we have confirmed that the equality always hold at the local point $(x^{(r)}[n],y^{(r)}[n],\mu^{(r)}[n])$. Therefore, the equality in (28) must hold at the same local point.

So far, the tight concave approximation for harvested power has been constructed. By replacing the harvested power with its concave approximation, we can obtain a convex approximation (P2) for the problem (P1)

$$
\begin{aligned}
\text{(P2)}: \max_{\{x[n],y[n],\mu[n]\},E} \quad & E && (29a)\\
\text{s.t.} \quad & \sum_{n=1}^{N} P^{(r)}_{k,n}(x[n],y[n],\mu[n])\delta T \geq E, \forall k\in\mathcal{K}, && (29b)\\
& (1),\ (13b).
\end{aligned}
$$

With the convex approximation (P2), the problem (P1) is then enabled to be solved in an iterative manner. Next, we will clarify our proposed iterative algorithm for problem (P1).

B. Proposed Iterative Solution

In the initialization step, i.e., $r=0$, we construct a feasible local point $(x^{(0)}[n],y^{(0)}[n],\mu^{(0)}[n])$. Then, in each iteration r, based on the local point $(x^{(r)}[n],y^{(r)}[n],\mu^{(r)}[n])$, we build the concave approximation $P^{(r)}_{k,n}(x[n],y[n],\mu[n])$ for the harvested power according to (28) and solve the convex problem (P2). After the the problem (P2) is optimally solved, the solution will be applied as the local point in the next (i.e., $r+1$-th) iteration. By repeating the iterations, the objective for problem (P1) will be constantly improved and eventually converges to a suboptimal point. Finally, from the solution, we can extract out the jointly optimized UAV trajectory $\{x[n],y[n]\}$ and directional antenna orientation scheme $\{\theta_0[n]\}$ according to $\theta_0[n]=\arccos\mu[n]$, $\forall n\in\mathcal{N}$.

IV. Simulation Results

Via simulations we evaluate our proposed algorithm. The average minimum harvested power within the charging period is treated as the key system performance indicator in the evaluations. The default parameter setups are provided as follows: First, the SNs are randomly distributed in a square area with width of 30m. Then, we set $K=5$, $H=5$m, $N_t=5$, $\beta_0=-30$dB, $P=40$dBm, $T=50$s and $V=1$m/s. Furthermore, the time resolution dT is set to $dT=1$s and the modification factor is chosen as $\delta=0.9$. Regarding the nonlinear EH model, a group of parameters are inherited from [14] and [22].

We first investigated 10 random topologies (of SN distributions) and display the average performance under varying duration T and different UAV speed limit V in Fig. 3. Note that the corresponding results of the case with an omni-directional antenna is also provided as a benchmark. In particular, the design for omni-directional antenna case can be obtained by simply assuming a constant antenna gain. With $\mathcal{F}_{\mathrm{A}}(\varphi_k[n])=1$, the convex approximation of the problem can be found based on the approximation in step (18). Then, an iterative algorithm can be similarly made to maximize the WPT performance for the omni-directional antenna case. From the figure, it can be found that for both cases with either a directional or an omni-directional antenna, a larger T and a larger V implies a better average WPT performance, i.e., minimum average harvested power among SNs. This is due to the fact that with a larger T, more time can be allocated for UAV staying at a position with a relatively higher WPT performance (possibly closer to the SNs). In addition, a larger V introduces a higher flexibility of UAV mobility, i.e., less time is spent for flying and more time can be used for hovering at a high performance position. Besides, we can also observe that the network with a directional antenna always outperforms that with an omni-directional antenna. As for reasons, the additional introduced antenna gain from the directional antenna has significantly compensated the pathloss in transmissions, so that more power can be wirelessly transferred with the optimized directional antenna orientation scheme.

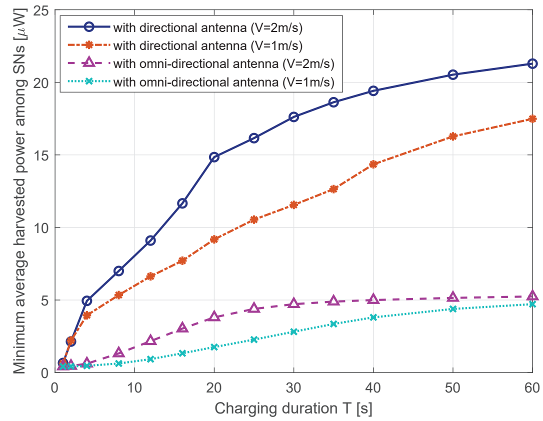

Fig. 3. WPT performance comparison with varying charging duration T.

Corresponding to Fig. 3, the trajectory difference between the directional antenna case and the omni-directional antenna case, and the directional antenna behaviour are provided in Fig. 4. First, from the trajectory comparisonwe find that the trajectory with a directional antenna tends to be closer to the SNs than that with an omni-directional antenna. This is due to the broadcasting channel property of the omni-directional antenna, under which the position closer to parts of the SNs does not necessarily result in a better (overall) performance, i.e., minimum average harvested power among SNs. By contrast,

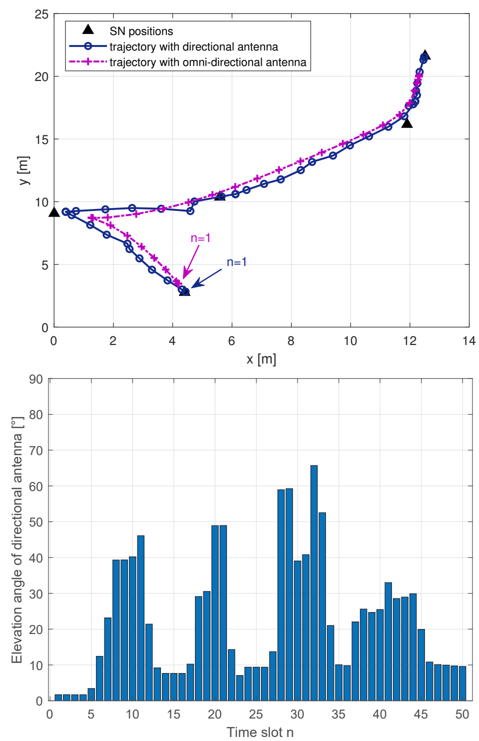

Fig. 4. Trajectory and directional antenna orientation ($V = 1$m/s, $T = 50$s).

with a directional antenna, at each time instant the UAV is able to (and highly likely does) cover not all but parts of the SNs, which makes it fly closer to the covered SNs and orient the directional antenna towards these SNs to gain a higher performance. From the observation of directional antenna orientation results, we find that when the horizontal distance between UAV and SNs are relatively larger, the optimized elevation angle turns to be larger to cover the SNs with main lobe. On the other hand, when the UAV is above a SN, the optimized elevation angle θ_0 approaches to zero to efficiently charge the targeted SN.

V. Conclusion

In this paper, we considered a UAV-enabled WPT network where a directional antenna (array) is deployed on the UAV. We studied the WPT performance of the network, i.e., the minimum harvested energy among all ground SNs, under an analog beamforming model of a ULA on the directional antenna and a nonlinear EH model. With a maximum speed limit and a given charging period, we considered a joint design aiming at maximizing the WPT performance via jointly optimizing the UAV trajectory and the elevation angle of the directional beamforming. To address the problem, we tightly approximate the harvested power to a concave lower-bound function, with which a tight convex approximation for the problem has been constructed. Finally, with our proposed algorithm, the problem is solved iteratively via efficiently handling a convex optimization problem in each iteration. Via simulations, by comparing to the case with omni-directional antenna, we have shown significant advantages of deploying a directional antenna in UAV-enabled WPT networks.

We finalize the whole paper by underlining the extensionality of the methodologies proposed in this work. This work for the first time proposed a joint optimization of directional antenna (directional beamforming) and UAV trajectory. The methodology of this combination highly likely assists also related joint designs under varied system assumptions.

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