Goodput Maximization in Slotted ALOHA Networks Operating with Finite Blocklength Codes

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***Abstract*—In this work, we consider an Internet of Things (IoT) system where slotted ALOHA is applied to handle the random access process of multiple IoT users. We leverage recent advances in the performance characterization in the finite blocklength (FBL) regime, and study the goodput and reliability performance of the network. In particular, we characterize the tradeoff between the transmission error due to FBL, the collision error influenced by FBL and user number, and the goodput. Following the characterization, an optimal system design is provided aiming at maximizing the goodput via choosing the optimal blocklength and determining the optimal number of users sharing the frequency resource of interests.**

***Index Terms*—collision, finite blocklength, Internet of Things, random access, slotted ALOHA**

I. Introduction

The support of future Internet of Things (IoT) scenarios calls for critical access control solutions for massive machine-type communications. For instance, the solutions are expected to handle wireless access requirements of a very large number of devices under low-latency constraints, while at the same time these devices usually require to be accessed, i.e., reporting information, less frequently. The traditional random access protocols [1], [2] find application in such scenarios, and have been considered as a promising solution. A key advantage of random access is that it does not require the massive devices to know the states of other devices, i.e., saving feedback overhead. The most thoroughly studied approach to random access is the slotted ALOHA due to the low complexity [3], [4]. To enhance the throughput, one promising approach is to shorten the slot length based on the average load [5].

However, all the above studies are conducted under the assumption of transmissions (when no collision occurs) being arbitrarily reliable at Shannon's capacity, which is only true in the so-called infinite blocklength regime (IBL). In particular, it is shown that in such IBL regime, reducing the blocklength of each transmission in the ALOHA process makes the probability of collisions be lower [6]. However, setting the blocklength to be short (no matter due to low-latency constraints or aiming to reduce the collisions) introduces a considerable decoding error probability to transmissions in the finite blocklength regime (FBL) [7]. In particular, as proved in [7] under the same condition (including coding rate and channel), a shorter blocklength leads to a higher decoding error probability. Moreover, for given frequency resources, if too many devices are allowed to share them in the ALOHA process, the collision probability will increase, while less devices also lead to a low total goodput or spectrum efficiency. In other words, while varying the blocklength of each ALOHA transmission and the number of users sharing the spectrum resource, there exists a tradeoff between the decoding error, collision and goodput. Hence, it is more essential to study the FBL performance while explicitly addressing the tradeoff in the FBL regime in the design of ALOHA networks. Nevertheless, recently the authors in [8] maximize the throughput of ALOHA in both IBL and FBL regimes via determining the optimal number of retransmissions. In addition, the energy efficiency of an ALOHA process within a limited frame length is improved in both IBL and FBL regimes by adjusting each user's transmission/access probability [9]. Moreover, the authors in [10] derive the FBL performance of all-to-all broadcasting slotted ALOHA within a limited frame length duration, where all the users are both transmitters and servers. To the best of our knowledge, it is still missing in the literature the optimal design in the FBL regime by addressing the tradeoff between the transmission error due to FBL, the collision error influenced by FBL and user number, and the goodput.

In this paper, we consider the impact of FBL on the process of slotted ALOHA and characterize the tradeoff between the transmission error, the collision and the goodput. An optimal system design is provided to maximize the goodput via deciding the optimal blocklength and determining the optimal number of users. Our contributions are summarized as follows:

- We characterize the impact of the blocklength on the system performance in terms of goodput and packet failure ratio (due to transmission error and collision).
- Following the characterization, we formulate a problem for the scheme design, which maximizes the goodput by optimizing the blocklength and the number of users under a packet failure ratio constraint. We optimally solve this non-convex problem in the following way: We rigorously prove that the goodput is pseudoconcave in the blocklength and is concave in the number of users. Then, the optimal solution is obtained via a proposed algorithm based on the modified block coordinate descent (BCD) method.

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- Via numerical analysis, we validate the analytical model and illustrate the tradeoff between decoding error, collisions and goodput, and investigate the influence of various parameters on the performance of the considered network. We also show the performance advantage of the proposed algorithm compared to the benchmark.

The rest of the paper is organized as follows. We describe our system in Section II. Then, we formulate and characterize the optimization problem and in Section IV, where the optimal algorithm addressing the problem is proposed as well. The simulation results are provided in Section V. Finally, we conclude our work in Section VI.

II. System Model

We consider an IoT network, where N user equipment (UEs) randomly access a server via shared radio frequency resources. The access process is operated in a slotted ALOHA manner, i.e., time is divided in successive slots with identical blocklength m (in symbols) and UEs are assumed to be synchronised with the server. Denote by T_S (in second) the time length of a single symbol. Therefore, the time duration (in second) of each time slot is $t = mT_S$.

We assume that the packet arrivals at different UEs are independent and identically distributed (i.i.d.), while the average arrive rate is μ (in packets/second) and each packet has k information bits. Therefore, the average network load G over all N users can be written as

$$G = \mu N t = \mu N T_S m. \tag{1}$$

Once a UE has a packet, it will send it over a whole time slot with a transmission rate $r = \frac{k}{m}$ (in bits/symbol). Since all UEs are synchronised, all the transmission requests of arrived packets in one slot (from different UEs) operate in the next slot. Note that under such ALOHA access mechanism, each UE has no knowledge of others. Hence, a collision may occur if more than one UE transmit in the same slot.

Assume that the channel state information (CSI) of a link is known by the corresponding UE. According to the CSI, the signal-to-noise ratios (SNRs) of all the potential transmissions (from different UEs) maintain at the same level γ at the server side, owning to power control mechanism [11].

III. Packet Error Probability and Goodput Characterization in the FBL Regime

A. Transmission Error Probability in FBL Regime

Due to the impact of finite blocklength, the transmission can be erroneous. In particular, with coding rate $\frac{k}{m}$, the transmission error probability in the ALOHA time slot is given by [7]:

$$\varepsilon_r = \mathcal{P}(\gamma, k, m) \approx Q\Big(\sqrt{\tfrac{m}{V(\gamma)}}(\mathcal{C}(\gamma) - \tfrac{k}{m})\log_e 2\Big), \tag{2}$$

where $\mathcal{C} = \log_2(1+\gamma)$ is the Shannon capacity. In addition, V is the channel dispersion [7]. Under a complex AWGN channel, $V = 1 - (1+\gamma)^{-2}$.

B. Collision Probability of a Transmitted Packet

As the UE has no knowledge of other UE's task arrivals, it is possible that multiple UEs choose the same time slot to transmit the data. Considering the task arrival of UE follows a Poisson distribution, the possibility of occupation with x UEs in the same time slot is given by [12]

$$P_{\text{occ}}(x) = \frac{G^x e^{-G}}{x!}. \tag{3}$$

Therefore, for a transmitted packet, the probability of colliding with other packet(s), given by ε_c, can be obtained based on (3). Note that ε_c is a conditioned probability on the event that there exists at least one transmitted packet. Hence, we have

$$\varepsilon_c = \frac{1 - P_{\text{occ}}(0) - P_{\text{occ}}(1)}{1 - P_{\text{occ}}(0)} = \frac{1 - e^{-G} - Ge^{-G}}{1 - e^{-G}}, \tag{4}$$

where $1 - P_{\text{occ}}(0)$ presents the probability that at least one packet is transmitted and $1 - P_{\text{occ}}(0) - P_{\text{occ}}(1)$ indicates the probability of collision.

C. Error Probability of a Transmitted Packet

Combining both the decoding error probability and the collision probability of a transmitted packet, the overall error probability of the packet is given by

$$\varepsilon_{\text{o}} = 1 - (1-\varepsilon_r)(1-\varepsilon_c) = 1 - \frac{(1-\varepsilon_r)Ge^{-G}}{1 - e^{-G}}. \tag{5}$$

D. Goodput

The overall goodput of the system is defined by the amount of information (in bits per second) which are successfully received at the server:

$$\tau_g = \frac{k(1 - P_{\text{occ}}(0))(1-\varepsilon_{\text{o}})}{t} = \mu N e^{-G} k (1-\varepsilon_r). \tag{6}$$

E. Packet Failure Ratio

Note that ε_{o} is the error probability conditioned on transmitting at least one packet in a slot. To characterize how many packets are failed to be received over the infinite time slots, the packet failure ratio can be obtained in the following way

$$P_F = \frac{G - \frac{t\tau_g}{k}}{G}. \tag{7}$$

IV. Problem Formulation

We aim at maximizing the goodput by deciding the optimal blocklength m of each time slot and determining the optimal number of UEs N sharing the resource of the ALOHA process. Therefore, the optimization problem is formulated as follows:

$$\underset{m, N}{\text{maximize}} \quad \tau_g \tag{8a}$$

$$\text{subject to} \quad P_{\text{F}} \le P_{\text{F,max}}, \tag{8b}$$

$$m \in \mathbb{N}, \tag{8c}$$

$$N \in \mathbb{N}. \tag{8d}$$

where constraint (8b) ensures that the packet failure ratio is less than a threshold $P_{\text{F,max}} \ll 1$ to prevent wasting radio resources.

Clearly, the problem is an integer non-concave problem with dual variables, due to the integer constraints (8d) and (8c), as well as the non-concave expression of objective. To tackle this difficulty, we solve the problem in the following three steps: We firstly decompose the original problem into two subproblems. Next, we characterize the subproblems, and subsequently we reformulate them into solvable relaxed pseudo-concave/concave problems. Finally, we propose an algorithm obtaining the optimal solution to the original problem by solving the subproblems iteratively via the BCD method.

A. Decomposition of Problem (8) *and Subproblem Characterization*

Letting N be fixed, i.e., $N = N^{\circ}$, we have following subproblem:

$$\begin{aligned} \underset{m}{\text{maximize}} \quad & \tau_g && \text{(9a)} \\ \text{subject to} \quad & N = N^{\circ} && \text{(9b)} \\ & \text{(8b) and (8c)} && \text{(9c)} \end{aligned}$$

By relaxing constraint (8c) as $m \geq 0$, we have following lemma to characterize the relaxed problem:

Lemma 1. *The relaxation of problem* (9) *is pseudoconcave in* $m \geq 0$.

Proof. We start to prove the objective function τ_g to be pseudoconcave. In particular, τ_g can be reformulated as

$$\tau_g = \frac{f(m)}{g(m)} \tag{10}$$

where

$$f(m) = \mu N k \left(1 - \varepsilon_r\right) \tag{11}$$

and

$$g(m) = e^{\mu N T_s m} . \tag{12}$$

On the one hand, the concavity of $f(m)$ can be showed by investigating the second order derivative:

$$\frac{d^2 f}{dm^2} = -\frac{d\varepsilon_r}{d\alpha}\frac{d^2\alpha}{dm^2} - \left(\frac{d\alpha}{dm}\right)\frac{d^2\varepsilon_r}{d\alpha^2} . \tag{13}$$

where $\alpha = \frac{C - \frac{k}{m}}{\sqrt{V/m}}$. With $\varepsilon_r = Q(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\infty} e^{-\frac{\alpha^2}{2}} d\alpha$ we have

$$\frac{d\varepsilon_r}{d\alpha} = -\frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}} \tag{14}$$

and

$$\frac{d^2\varepsilon_r}{d\alpha^2} = \frac{\alpha}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}} . \tag{15}$$

This results in the second derivative of ε as follows:

$$\frac{d^2\varepsilon_r}{dm^2} = \underbrace{\frac{1}{\sqrt{2\pi}} e^{-\frac{\alpha^2}{2}}}_{\geq 0} \left(\alpha \underbrace{\left(\frac{d\alpha}{dm}\right)^2}_{\geq 0} - \frac{d^2\alpha}{dm^2} \right) \tag{16}$$

Therefore, the sign of $\frac{d^2\varepsilon}{dm^2}$ depends on $-\frac{d^2\alpha}{dm^2}$, which can be written as:

$$-\frac{d^2\alpha}{dm^2} = \frac{C + \frac{k}{m}}{4m^2\sqrt{\frac{V}{m}}} \geq 0 . \tag{17}$$

If the transmission rate is lower than Shannon capacity, i.e., $C - \frac{k}{m} \geq 0$, we have $\alpha \geq 0$. Hence, it holds that $\frac{d^2 f}{dm^2} \leq 0$. If the transmission rate is higher than Shannon capacity, i.e., $C - \frac{k}{m} \leq 0$, we have $\varepsilon_r \geq 1/2$ so that $P_F \geq 1/2$, which violates the constraint (8b) and leads to an infeasible solution of (9). As a result, it holds that $\frac{d^2 f}{dm^2} \leq 0$, i.e., $f(m)$ is concave w.r.t. m.

On the other hand, $g(m) = e^{\mu N T_S m}$ is an exponential function w.r.t m and therefore convex and strictly positive.

Based on [13, Lemma 2.1], $\tau_g = \frac{f(m)}{g(m)}$ is pseudoconcave, since (i) $f(m)$ is non-negative concave and (ii) $f(m)$ is strictly positive convex.

Next, we move on to discussing P_{F}. Similarly, we also reformulate P_{F} as

$$P_F = 1 - \frac{\tilde{f}(m)}{g(m)} \tag{18}$$

where $\tilde{f}(m) = (1 - \varepsilon_r)$. As showed before, $\frac{\tilde{f}(m)}{g(m)}$ is also a pseudoconcave function. Therefore, $P_F = 1 - \frac{\tilde{f}(m)}{g(m)}$ is pseudoconvex w.r.t. m.

In summary, problem (9) is a pseudoconcave problem. □

Next, by letting m be fixed, i.e., $m = m^{\circ}$, we have the second subproblem:

$$\begin{aligned} \underset{N}{\text{maximize}} \quad & \tau_g && \text{(19a)} \\ \text{subject to} \quad & m = m^{\circ} && \text{(19b)} \\ & \text{(8b) and (8d)} && \text{(19c)} \end{aligned}$$

By relaxing constraint (8d) as $N \geq 0$, we have following key lemma to address the subproblem:

Lemma 2. *The relaxation of problem* (19) *is concave in* $N \geq 0$.

Proof. We also start with the objective function τ_g. The concavity of τ_g can be showed by second order derivative:

$$\frac{\partial^2 \tau_g}{\partial N^2} = \mu \left(1 - \varepsilon_r\right) m T_s \left(G - 2\right) e^{-G} . \tag{20}$$

If $G \leq 2$, we have $\frac{\partial^2 \tau_g}{\partial N^2} \leq 0$. If $G > 2$, we have $P_F \geq \frac{2 - 2e^{-2}(1-0)}{2} \geq 0.86 \gg P_{F,\max}$, where N is infeasible. Therefore, a feasible solution of N must hold that $G = \mu N m \leq 2$.

Next, we investigate the convexity of P_F w.r.t N. The second derivative of P_F is given by

$$\frac{\partial^2 P_F}{\partial N^2} = (\mu N T_S m)^2 e^{-G} \geq 0 \tag{21}$$

Hence, P_F is convex in N. Since τ_g is concave in N and P_F is convex in N, problem (19) is a convex problem w.r.t.

N. □

According to Lemma 1 and 2, subproblems (9) and (19) can be solved efficiently by applying standard convex optimization methods, respectively.

B. Optimal Solution to Problem (8)

To obtain the solution of the original problem, we propose a modified BCD [14] algorithm, where we fix each one of the variables and obtain the optimal solution for another successively. In particular, by initiating $N^{(0)} = N^{\circ}$ and $m^{(0)} = m^{\circ}$, we firstly solve problem (9) with $N = N^{(i)}$ to obtain $m^{(i+1)} = \underset{m}{argmax}\ \tau_g(m, N^{(i)})$, then solve problem (19) with $m = m^{(i+1)}$ to obtain $N^{(i+1)} = \underset{N}{argmax}\ \tau_g(m^{(i+1)}, N)$ in each iteration. Those steps are repeated until the solution converges, where an optimum tuple $(m^{(I)}, N^{(I)})$ is obtained after I iterations. According to [14], the algorithm converges to the global optimum with a sub-linear convergence rate.

Note that the tuple $(m^{(I)}, N^{(I)})$ is based on the relaxation of the problem. To obtain the integer solution, we have to compare all possible integer neighbors of $(m^{(I)}, N^{(I)})$ and choose the solutions that maximize the goodput while fulfilling the constraint (8b) as the optimal solution (m^*, N^*). The algorithm is outlined in Algorithm 1.

Algorithm 1 Search Algorithm for Joint Optimum

Solve relaxed problem:
Initialize $N^{(0)} = N^{\circ}, m^{(0)} = m^{\circ}, i = 1$
while solution does not converge **do**
  Solve $\max_{m} \tau_g(m, N^{(i-1)})$ according to Lemma 1, obtain $m^{(i)}$
  Solve $\max_{N} \tau_g(m^{(i)}, N)$ according to Lemma 2, obtain $N^{(i)}$
  Start next iteration $i = i + 1$
end while
Non-integer solution is $(m^{(I)}, N^{(I)}) = (m^{(i)}, N^{(i)})$

Solve original problem:
Find corner points of $(m^{(I)}, N^{(I)})$:
Find $(m^*, N^*)_1 = \arg \max_{m; N \in \{\lfloor N^{(I)} \rfloor, \lceil N^{(I)} \rceil\}} \tau_g(m, N)$
Find $(m^*, N^*)_2 = \arg \max_{m \in \{\lfloor m^{(I)} \rfloor, \lceil m^{(I)} \rceil\}, N} \tau_g(m, N)$
Choose tuple with maximal goodput:
$(m^*, N^*) = \arg \max_{(m,N) \in \{(m^*,N^*)_1, (m^*,N^*)_2\}} \tau_g(m, N)$

The optimal solutions are (m^*, N^*)

V. Numerical Simulations

In this section, we validate our analytical model via Monte Carlo simulations and show the advantage of our proposed algorithm in comparison to benchmarks. In the simulations we consider the following parameter setups: We have $N = 600$ users with an arrival rate of $\mu = 1000$ packets per second. Each packet contains $k = 256$ bits, encrypted in symbols with

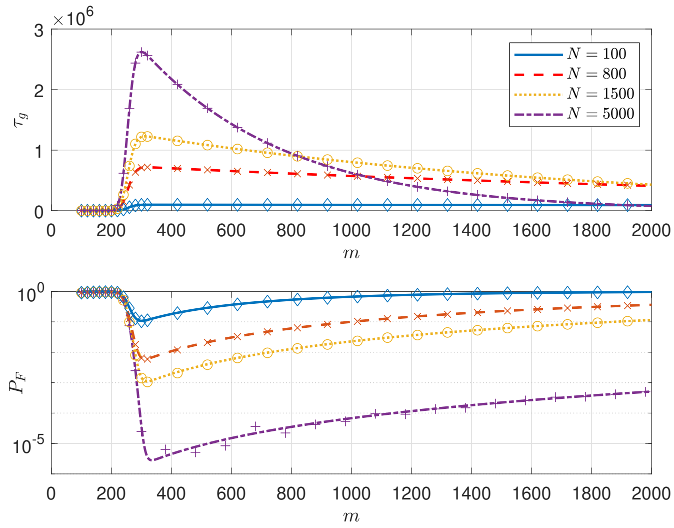

Fig. 1. Influence of the user number on goodput τ_g and packet failure ratio P_F. The system setup is the following: various numbers of users with an arrival rate of $\mu = 1000$ packets per second, SNR $\gamma = 1$, $k = 256$ bits per packet, $T_s = 2.4 \times 10^{-9}$ per second.

symbol duration $T_s = 2.4 \times 10^{-9}$ s. The SNR level at server side is set as $\gamma = 1$.

First, we investigate how the blocklength influences the goodput τ_g and error probability P_F. The results are provided in Fig. 1, where different setups of user number N are considered. In addition, both the numerical results based on our analytical model (lines) and Monte Carlo simulation results (markers) are provided. First of all, the simulation results match with our analytical model. Second, the results in Fig. 1-up confirm our Lemma 1 that the goodput τ_g is pseudoconcave in m. In particular, we can observe that the optimal choices of m are located in a relatively short bocklength regime. This further confirms the necessity of considering the FBL impact in the system design. On the other hand, when a longer blocklength m is chosen, the system performance decreases rapidly, especially when N is large. Hence, the results suggest that an accurate optimal value of m is more important for large slotted ALOHA-supported IoT networks. Third, the results provided in the bottom subplot of Fig. 1 validate our discussion after (18) that the error probability of a transmitted packet is pseudoconcave in m. In addition, the optimal choices of m minimizing the error probability P_F matches with that for maximizing τ_g. More interestingly, if we aim at minimizing P_F, a shorter N is always preferred, i.e., no matter if m is short or long.

Recall that in our design, we optimally choose the blocklength m and the number of users N. Hence, we investigate the joint impact of m and N on the goodput τ_g in Fig. 2. Overall, τ_g shows joint psuedoconcavity w.r.t. N and m, which ensures that the optimal value can be obtained via the proposed BCD method. Moreover, due to the error probability constraints, the problem is not always feasible if N is too high, or m is too short. The envelope, which separates the regime into

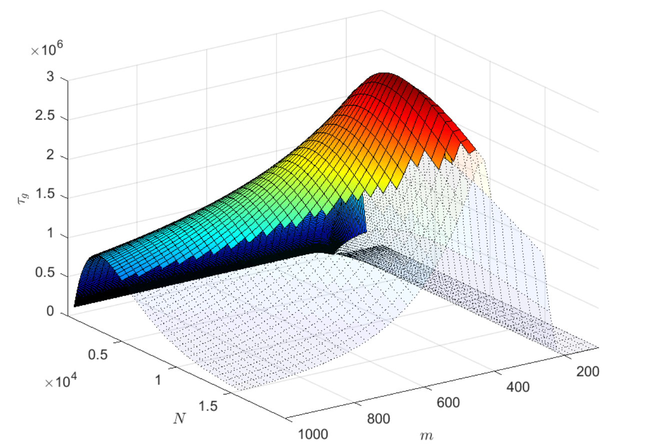

Fig. 2. Joint influence of user number N and blocklength m on goodput τ_g and packet failure ratio P_F The system setup is the following: various numbers of users with an arrival rate of $\mu = 1000$ packets per second, SNR $\gamma = 1$, $k = 256$ bits per packet, $T_s = 2.4 \times 10^{-9}$ s, $P_{\mathrm{F,max}} = 0.001$.

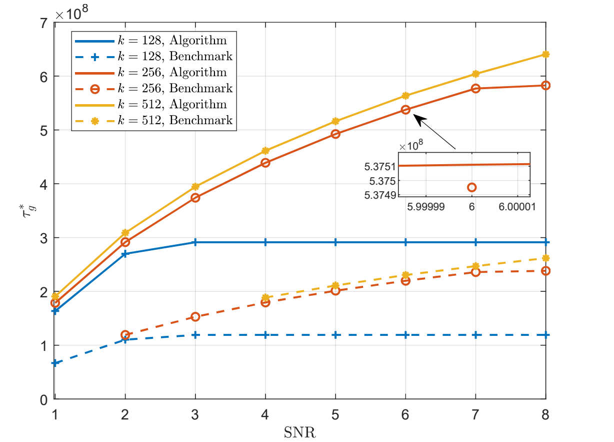

Fig. 3. Optimal goodput τ_g^* versus SNR with proposed algorithm and benchmark. The marker implies the optimal goodput with relaxed constraints while the lines represent the optimal goodput with integer constraints. The system setup is the following: various numbers of packet size k with an arrival rate of $\mu = 1000$ packets per second, $T_s = 2.4 \times 10^{-9}$ s, $P_{\mathrm{F,max}} = 0.001$.

the feasible set (solid surface) and the infeasible set (faded surface), satisfies $P_F = P_{F,\max}$. It confirms our Lemma 1 and 2 that the feasible set, which is limited by $P_{F,\max}$, is a convex set.

Finally, in Fig. 3 we show the relationship between the optimal goodput, i.e., obtained via the proposed algorithm, versus the received SNR level at the sever side. Different cases with variant packet size k are considered. In addition, the results are plotted as lines/curves, while the goodput based on the optimal solution of the relaxed problem is represented by markers as a performance reference. Moreover, the FBL goodput based on the optimal solution obtained in the IBL regime is also provided as the benchmark, this would help to show the performance loss of ignoring the FBL impact. In particular, it has been shown that in the IBL regime the maximum goodput is reached when the average load $G = 1$ [12], since the transmission is error-free as long as the transmission rate is lower than Shannon's Capacity. Therefore, we set $N_{\text{benchmark}} = N^*$ and $m_{\text{benchmark}} = 1/\mu T_S N^*$ to obtain this benchmark goodput.

First, we observe that all goodput curves are increasing in SNR. The speed of the increase is high in the low SNR region. But it becomes slow in the high SNR regime, especially when the packet size is small. In such case, the bottleneck of the system becomes the collision, as the SNR only influences the transmission error probability ε_r. Therefore, purely increasing SNR does not always make sense for goodput promotion. Second, as shown in the figure, the performance with the optimal integer solutions matches well with the optimal performance of the corresponding relaxed problem. In the zoom-in subplot, we show that the performance gap is tiny. Third, the proposed algorithm significantly outperforms the benchmark (IBL) when the same setups are considered. In addition, the optimal solution based on the IBL assumption is not always feasible w.r.t. the error probability constraint. These observations confirm again the necessity of considering the FBL impact in the design of the considered ALOHA system.

VI. Conclusion

In this work, we consider slotted ALOHA-supported IoT system operating with short blocklengths. We leverage recent advances in the performance characterization in the FBL regime, and study the goodput and reliability performance of the network. In particular, we characterize the tradeoff between the transmission error, the collision error, and the goodput. Moreover, we rigorously prove that the goodput is pseudoconcave in the blocklength and is concave in the number of users. Following the characterization, an optimal system design is provided aiming at maximizing the goodput via deciding the optimal blocklength and determining the optimal number of users, while the optimal solution is obtained via a proposed algorithm based on the modified BCD method. Via numerical analysis, we validate the analytical model and investigate the system performance. In particular, the results show that a relatively short blocklength is preferred both in maximizing the goodput or minimizing the error probability. In addition, a significant performance loss is observed if the FBL impact is ignored. Both of them confirm the necessity of our design in the FBL regime.

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